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# Predicting Sovereign Fiscal Crises: High-Debt Developed Countries

Betty C. Daniel, Department of Economics, University at Albany – SUNY
and
Christos Shiamptanis, Department of Economics, Wilfrid Laurier University

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Betty C. Daniel
Department of Economics
University at Albany - SUNY

Christos Shiamptanis*
Department of Economics
Wilfrid Laurier University

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**Abstract**

Every country has a fiscal limit on debt, beyond which the country's economic and political systems cannot raise taxes and/or reduce spending sufficiently to maintain solvency. We assume fiscal crises are created by insolvency due to fiscal limits, and use historical data on debt and surpluses to explain why countries with similar debt levels have different crisis experiences. We use values for the maximum historical surplus together with estimates of fiscal feedback rules to estimate debt limits for each country. Debt limits vary considerably across countries. Defining fiscal space as the largest increase in debt, for a particular surplus, consistent with solvency, we separate countries into risk categories based on fiscal space. Greece and Portugal eroded their fiscal space several years, prior to their fiscal crises, placing them in the highest risk category and predicting the crises that followed. Canada and Belgium maintained large enough fiscal space to achieve safe status. Other countries reduced fiscal space, with three eroding fiscal space in 2014, warning of future crises.

**JEL Classifications:** E62, F34

**Keywords:** Fiscal Limits, Fiscal Space, Fiscal Rules, Fiscal Solvency, Sovereign Default

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# 1 Introduction

Greece lost access to credit markets when debt exceeded 130 percent of GDP. In contrast, Belgium successfully retained market access with similar debt relative to GDP. What determines the value of debt beyond which a country looses access to financial markets? Why do countries exhibit heterogeneity in debt tolerance? We focus on ten high-debt, developed countries, two of which lost access to markets after the 2008 global financial crisis. The high-debt countries include Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, the US, and the UK.

We assume that fiscal crises are due to insolvency created by fiscal limits (Bi (2012), Davig, Leeper and Walker (2011)). Every country faces a limit on the value of the primary surplus relative to GDP that it can raise to repay debt. If debt repayments require larger current and expected future values for the surplus, then the sovereign is insolvent. Creditors refuse to lend into a position of insolvency, creating a fiscal crisis. We develop a model, using historical data on primary surpluses and debt, both relative to GDP, to explain why countries with similar debt levels have different experiences with crises. Additionally, we use the model to place high-debt countries into three risk categories, safe, risky, highest risk, in each year following the 2008 financial crisis.

We follow Tanner (2013) and Collard et al. (2015) and use historical experience to identify a maximum primary surplus relative to GDP, which we use as an estimate of the surplus limit. However, we depart from their work in a key way. We assume that this maximum value cannot be attained instantaneously and maintained forever. Raising the surplus to a new value after shocks could take time, possibly due to the political process, implying that the maximum surplus is not reached immediately. Additionally, even if the surplus limit is a technical concept, like the top of the Laffer curve, a government might not have the political will to maintain it forever. For example, Greece agreed to raise its surplus to the 3.5 percent demanded by the Troika, but stated that it cannot maintain this value indefinitely. We replace the assumption, that a government can instantaneously and forever attain the maximum surplus, with the assumption that the government adjusts its surplus according to a fiscal feedback rule, relating the current surplus to its own lag and to lagged debt.[1]

For each of our ten countries, we estimate fiscal feedback rules for the primary surplus

[1] Celasun, Debrun, and Ostry (2007) also use a fiscal feedback rule in assessing fiscal risk. They interact their estimate of the rule with the value of expected future shocks based on an estimated VAR, to predict probability bounds on future values of debt. Debt is viewed as non-sustainable if these probability bounds indicate rising debt.

and use them to estimate country-specific debt limits. The estimated parameters of the fiscal feedback rule predict paths for future surpluses and debt conditional on initial debt-surplus pairs. The path with the peak value of the surplus at the surplus limit contains the largest possible future surpluses. Therefore, the debt limit, is the peak value of debt along this path. We find considerable heterogeneity in debt limits across countries with Japan, Belgium, and Canada having high limits and Portugal having the lowest. These debt limits are important determinants of debt tolerance, partly explaining why countries with similar levels of debt have different crisis experiences.

We use each country's current debt-surplus pair, together with its debt limit and expected future path for the surplus and debt, to place countries into risk categories. First, we use the expected adjustment path, with peaks at the surplus and debt limits, to construct a boundary path for debt-surplus pairs which separates them by solvency. Positions for debt along the boundary path represent the highest value for debt consistent with solvency. A fiscal crisis occurs when an adverse shock, together with default expectations, sends the debt-surplus pair above its boundary path. Creditors refuse to lend, forcing default to restore a position of solvency along the boundary path. Second, we compute fiscal space, defined as the difference between the current value of debt and its value along the boundary path for a particular surplus. A larger surplus generally implies a higher position of debt along the boundary path and therefore larger fiscal space.[2]

Third, fiscal space, together with the probability distribution of shocks, allows us to categorize a country's risk of a one-period-ahead fiscal crisis. Countries with low debt relative to the boundary path, have large fiscal space and low risk. We label a country as safe if fiscal space is so large that the probability of receiving shocks large enough to send debt above the boundary path next period is virtually zero. Alternatively, the country is at highest risk if it has exhausted its fiscal space. Intermediate values of fiscal space place countries at risk, with risk increasing at an increasing rate as fiscal space shrinks (Daniel and Shiamptanis 2012). We compute measures of fiscal space for each of our ten high-debt developed countries for each year following the 2008 global financial crisis, and use this measure to place countries in risk categories.

Greece and Portugal begin the period in the highest risk category, having exhausted fiscal space. Portugal succumbed to crisis three years later, even though it had lower debt than all the other countries, and Greece succumbed two years later with debt lower than that in Belgium, Canada, Italy and Japan. For both countries, several years of zero fiscal

[2]This result is consistent with empirical evidence (Chakrabarti and Zeaiter 2014). Once the surplus gets high enough, an increase no longer increases fiscal space as explained in Section 2.4.

space put them in the highest risk category, predicting the crises they experienced. In contrast, Canada and Belgium sustain large enough fiscal space to maintain safe status throughout, even though Belgium is the second highest debt country after Japan. The UK and Italy experience falling fiscal space and therefore rising risk throughout the period, but never exhaust fiscal space. Other countries, including the US, do exhaust their fiscal space for some years. France, Spain, and Japan have no fiscal space in 2014, placing them in the highest risk category and warning of possible future fiscal crises.

Our work builds on that of others who have used fiscal limits to assess insolvency risk. Bi (2012) assumes that there is a fiscal limit on the value of the surplus determined by the value of tax revenue at the top of the Laffer curve, which in turn implies a fiscal limit on debt. She finds that countries further down their Laffer curves are less risky because they are further from their fiscal limit. Ghosh et al. (2013) offer a measure of a debt limit based on the assumption that future surpluses are determined by a nonlinear function of powers of debt. In their panel model, the cubic power of debt has a negative coefficient, enabling them to use country-specific interest rates to identify country-specific values of debt beyond which debt explodes. Since no creditor would lend to a borrower whose debt is explosive, the debt limit is the value of debt at which the system becomes explosive.[3] Tanner (2013) identifies an upper bound on the surplus and infers a debt limit from the government's intertemporal budget constraint as the present value of the maximum surplus. Collard et al.(2015) modify the Tanner analysis using an upper bound on the surplus, adjusted for the probability that a country defaults on debt. They calculate maximum sustainable debt as the value of debt beyond which debt becomes explosive due to its default premium.

This paper is organized as follows. The next section derives the debt limit and fiscal space. Section 3 is empirical, with estimates of fiscal feedback rules. Section 4 uses our estimates to measure fiscal space and assess the risk of a fiscal crisis following the global financial crisis for our ten high-debt developed countries. Section 5 provides conclusions.

[3] This measure requires the assumption that responsiveness to debt continues to fall as debt rises to its limit, a value well outside of values of debt in the sample. Debt limits cannot be identified if the coefficient on the cubic power of debt is not negative, or if a country's interest rate implies instability over all values of debt.

# 2 Fiscal Limits and Fiscal Solvency

## 2.1 Surplus Limit

Every sovereign faces a limit on its ability to raise government surpluses with tax increases and spending reductions, and therefore a limit on its ability to repay and service debt. Davig et al. (2011), and Bi (2012) motivate the surplus limit by the top of the Laffer curve for distortionary taxes. However, the concept can be more general. A limit on the surplus can be due to the inability to reduce government spending, perhaps due to the dependence of economic activity on the provision of public goods, and to the inability to raise tax rates for other reasons, including tax evasion (Daniel 2014). Bi et al. (2013) argue that the surplus limit could also be political, whereby the democratic process cannot raise the surplus sufficiently to service the debt. We denote the absolute largest value of the primary surplus relative to GDP as $\bar{s}$, and express the surplus limit by

$$s_t \leq \bar{s} \qquad \forall t. \tag{1}$$

## 2.2 Debt Limit

A surplus limit also implies a debt limit through the government's budget constraint. The government's flow budget constraint, with the debt ($d_t$) and primary surplus ($s_t$) both expressed as a fraction of GDP, is given by

$$d_t = \delta_t \left( \frac{1 + i_{t-1}}{1 + \rho_t} \right) d_{t-1} - s_t, \tag{2}$$

where $\delta_t$ denotes the fraction of debt that is repaid, $i_{t-1}$ is the domestic interest rate, $\rho_t$ is the growth rate of domestic output and $\frac{1+i_{t-1}}{1+\rho_t}$ has the interpretation as the domestic growth-adjusted interest rate.[4]

We use interest rate parity to express the evolution of debt as a linear function of the surplus, lagged debt, and stochastic terms. First, assume that the value of the domestic interest rate is determined by interest rate parity, where, as a simplification, we assume that the foreign interest rate ($i$) is fixed, yielding

$$1 + i = (1 + i_{t-1}) \, E_{t-1} \delta_t. \tag{3}$$

Expectations of default ($E_{t-1}\delta_t < 1$) require the domestic interest rate ($i_{t-1}$) to rise to

[4] We can view $i_t$ and $\rho_t$ either as nominal or real with no effect on the derivation.

offer a risk-neutral investor an expected rate of return equal to that in the market.

Assume that the inverse of the gross domestic growth rate $\left(\frac{1}{1+\rho_t}\right)$ is distributed iid about a mean of $\frac{1}{1+\rho}$ such that

$$\frac{1}{1+\rho_t} = \frac{1}{1+\rho}\zeta_t, \tag{4}$$

where $\zeta_t$ captures stochastic growth shocks and $E_{t-1}\zeta_t = 1$. Using this assumption, define interest rates adjusted by the mean domestic growth rate as

$$(1+r) = \frac{1+i}{1+\rho}, \qquad (1+r_{t-1}) = \frac{1+i_{t-1}}{1+\rho}. \tag{5}$$

Using equations (4) and (5), the equation for the evolution of debt (2) becomes

$$d_t = \zeta_t \delta_t \left(1+r_{t-1}\right) d_{t-1} - s_t. \tag{6}$$

Substituting equation (5) into interest rate parity, equation (3), and dividing both sides by $1+\rho$ yields

$$1+r = \left(1+r_{t-1}\right) E_{t-1}\delta_t. \tag{7}$$

Define $\alpha_t$ as

$$\alpha_t = \left(1-\delta_t\right)\left(1+r_{t-1}\right) d_{t-1}, \tag{8}$$

where $\alpha_t$ has the interpretation as the capital loss due to default. Using equation (8) and substituting from equation (7), unexpected capital loss due to default can be expressed as

$$\alpha_t - E_{t-1}\alpha_t = \left[-\delta_t \left(1+r_{t-1}\right) d_{t-1} + \left(1+r\right) d_{t-1}\right]. \tag{9}$$

Substituting $\delta_t \left(1+r_{t-1}\right) d_{t-1}$ from equation (9) into the equation (6) yields

$$d_t = \left(1+r\right) d_{t-1} - s_t - \eta_t + \psi_t. \tag{10}$$

where

$$\psi_t = \left[\zeta_t - 1\right]\left(1+r\right) d_{t-1}, \tag{11}$$

$$\eta_t = \left[\alpha_t - E_{t-1}\alpha_t\right]\zeta_t. \tag{12}$$

Equation (10) is an equation for the evolution of debt which is linear in lagged debt, the surplus, and mean-zero stochastic terms. An adverse growth shock ($\zeta_t > 1$) raises the growth-adjusted interest rate, increasing $\psi_t$, thereby raising debt. The expectation of capital loss due to default ($E_{t-1}\alpha_t > 0$) raises debt by raising the domestic interest rate to include a default premium, whereas actual capital loss due to default, ($\alpha_t > 0$), reduces debt. When the expectation of default equals actual default, there is no debt reduction ($\eta_t = 0$) because the interest rate fully adjusts to offset the future default.

Solving equation (10) forward for debt and imposing $\lim_{J\longrightarrow\infty} \frac{d_{t+J}}{(1+r)^J} = 0$ yields an expression for the government's intertemporal budget constraint,[5]

$$d_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j+1} \left[s_{t+1+j} + \eta_{t+1+j} + \psi_{t+1+j}\right]. \tag{13}$$

Satisfaction of the government's intertemporal budget constraint, equation (13), does not require that the government never default, or equivalently that $\alpha_{t+1+j} = 0 \ \forall j$. Default can occur, but it provides revenue only if it is larger than its expected value yielding $\eta_{t+1+j} > 0$. Therefore, systematic and expected default cannot provide revenue.

The expected intertemporal budget constraint is the expectation of equation (13)[6],

$$d_t = E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j+1} s_{t+1+j}, \tag{14}$$

validating that default cannot generate expected (and systematic) revenue. A solvent government must be expected to generate future surpluses whose present value equals the value of debt. The surplus limit adds a constraint on expected future surpluses and therefore on debt.

To specify the debt limit, we must determine the maximum of the expected present value of surpluses, given that no surplus can exceed $\bar{s}$. If a country could obtain its absolute largest surplus every period, then the debt limit would be the present value of the surplus limit, or equivalently[7]

$$d_t \leq \bar{d}_t = \frac{\bar{s}}{r}. \tag{15}$$

However, it is possible that a country could reach $\bar{s}$ only with a lag and/or maintain it for a limited period of time. In these cases, the debt limit depends on the set of largest

[5] The government's no Ponzi game constraint rules out a positive value. The no Ponzi game constraint for the household (or the aggregate of the remaining agents in the market) rules out a negative value.

[6] $E_t\eta_{t+1+j} = E_t\left(\zeta_{t+1+j}\left[\alpha_{t+1+j} - E_{t+j}\alpha_{t+1+j}\right]\right) = E_t\left(\zeta_{t+1+j}\alpha_{t+1+j}\right) - E_t\left(\zeta_{t+1+j}E_{t+j}\alpha_{t+1+j}\right) = E_t\left(\zeta_{t+1+j}\alpha_{t+1+j}\right) - E_t\left(\zeta_{t+1+j}\alpha_{t+1+j}\right) = 0$

[7] The debt limit is the largest debt the country could borrow, not maximum sustainable debt (Collard et al 2015).

surplus values across time that the sovereign could maintain, yielding

$$d_t \leq \bar{d}_t = \max \left\{ E_t \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^{j+1} s_{t+1+j} \right\} \leq \frac{\bar{s}}{r} \tag{16}$$

Determination of the debt limit requires prediction of the largest future surpluses.

## 2.3 Fiscal Feedback Rule

To predict future surpluses and debt, we follow Bohn (1998, 2007) and assume that the government follows a fiscal feedback rule in which the surplus systematically reacts to economic variables. Consider a benchmark specification, in which the surplus responds to both to its own lag and to lagged debt $(d_{t-1})$ according to

$$s_t = c + \beta s_{t-1} + \gamma d_{t-1} + \epsilon_t. \tag{17}$$

The parameter $c$ is a constant governing the long-run value of the surplus, and $\epsilon_t$ represents zero-mean stochastic shocks, due both to the policy process and to business cycles. We allow the surplus to exhibit persistence by including its lag and assume that $0 \leq \beta < 1$, consistent with our empirical evidence. The parameter $\beta$ captures the inertia in the legislative process, and the parameter $\gamma$ captures the government's responsiveness to debt. The fiscal feedback rule, equation (17), together with the government's flow budget constraint, equation (10), predict paths for future debt and surpluses.

The requirement that expected future debt and surpluses remain bounded by limits implies that the dynamic system in expectations of debt and the surplus, given by expectations of equations (10) and (17), be dynamically stable. Taking equations (10) and (17) $j$ periods forward and taking the time $t$ expectation yields

$$E_t d_{t+j} = (1+r) E_t d_{t+j-1} - E_t s_{t+j}, \tag{18}$$

$$E_t s_{t+j} = c + \beta E_t s_{t+j-1} + \gamma E_t d_{t+j-1}, \tag{19}$$

where $E_t \eta_{t+j} = E_t \psi_{t+j} = E_t \epsilon_{t+j} = 0$. The empirically relevant restriction for dynamic stability of the system becomes[8]

$$\gamma > r(1-\beta). \tag{20}$$

[8]Dynamic stability requires that the roots $(\theta)$ of the characteristic equation given by, $\theta^2 - \theta(1+r-\gamma+\beta) + \beta(1+r) = 0$, be within the unit circle. Therefore, their product, $\beta(1+r)$, be less than one. Given this restriction, stability requires equation (20).

If the system in the debt and surplus were not dynamically stable, then the expectations for the debt and the surplus would be unbounded over time, eventually exceeding any fiscal limit. This is a position of insolvency into which agents would not lend. Therefore, the ability to borrow requires parameter values consistent with stability.

Dropping time subscripts in equations (18) and (19) and solving for debt and the surplus yields long-run values as

$$d = \frac{-c}{\gamma - r(1-\beta)}, \qquad s = \frac{-rc}{\gamma - r(1-\beta)}. \tag{21}$$

We use a phase diagram to illustrate the expected future values of the surplus and debt over time for given initial values. To derive the equations for the phase diagram, subtract the lagged values of the left-hand side variable from both sides of equations (18) and (19) and set expected changes in each variable to zero to yield

$$E_t d_{t+j-1}|_{(\Delta E_t d_{t+j}=0)} = \frac{c + \beta E_t s_{t+j-1}}{r - \gamma}, \tag{22}$$

$$E_t d_{t+j-1}|_{(\Delta E_t s_{t+j}=0)} = \frac{-c + (1-\beta) E_t s_{t+j-1}}{\gamma}. \tag{23}$$

For $\gamma > r$, the $\Delta E_t d_{t+j} = 0$ has a negative slope,[9] and the $\Delta E_t s_{t+j} = 0$ curve has a positive slope. Arrows of motion for both curves point toward each curve, confirming a globally stable model.

## 2.4 Boundary Path

We use predicted values of the debt and surplus to construct a boundary path which separates debt-surplus pairs by solvency. Figure 1 contains the phase diagram. Point E represents long-run equilibrium values for debt and the surplus, given by equations (21). Adjustment paths ACE and BWE reflect paths for expected future values of the surplus and debt for different initial values of the debt-surplus pair.

The highest possible future surpluses must be consistent with the fiscal feedback rule and with a maximum surplus of $\bar{s}$. Therefore, the expected future adjustment path which peaks at $\bar{s}$, labeled BWE, contains the set of highest possible future surpluses. This implies that the debt limit, $\bar{d}$, is the peak for debt along this path at point W.

We use Figure 1 to determine the boundary path. For an initial debt-surplus pair on

[9] When $\Delta E_t d_{t+j} = 0$ has a positive slope, stability requires that it be steeper than $\Delta E_t s_{t+j} = 0$, implying a similar pattern for arrows of motion.

or below BWE, expected future values of the debt and the surplus do not exceed fiscal limits, implying solvency. At the other extreme, if the country's debt is above the debt limit, then it is insolvent.[10]

Consider the case where shocks have sent the debt-surplus pair above BW, but below $\bar{d}$. For these positions, expected future values of debt violate the debt limit. To avoid violation, the country would need higher surpluses, an impossibility. Therefore, these position are not consistent with solvency. Finally, consider initial debt-surplus pairs below $\bar{d}$, but above WE. If the country follows its fiscal feedback rule, then debt is not expected to violate the fiscal limit, but the surplus is. However, the adjustment path represents maximum surpluses, and the country could choose to raise its surplus less aggressively, such that it is expected to reach a long-run equilibrium without violating either fiscal limit. Therefore, these positions are also solvent.[11]

The boundary path, separating solvent from insolvent positions is given by BWZ. Any debt-surplus pair above BWZ is insolvent. Default in the magnitude necessary for the debt-surplus pair to reach the boundary path restores solvency and lending.

## 2.5 Fiscal Space

We define fiscal space as the largest one-period-ahead increase in debt for which the country is expected to remain solvent, equivalently remain below the boundary path BWZ. If the economy begins at debt-surplus pair A, then it is expected to transition along the adjustment path AE to point C.[12] Fiscal space is the maximum increase in debt from point C, subject to the constraint that debt not be above BWZ, following adverse changes in the exogenous shocks $\left(\psi_{t+1}, \epsilon_{t+1}\right)$, or in the endogenous value of $\eta_{t+1}$ due to expected default $\left(\eta_{t+1} = -\zeta_{t+1} E_t \alpha_{t+1} < 0\right)$.[13] Fiscal space must be greater than or equal to zero to justify lending.

Either an increase in expected default $\left(\eta_{t+1} < 0\right)$, or an adverse debt shock $\left(\psi_{t+1} > 0\right)$ would raise debt with an unchanged surplus. The vertical distance between point C and the boundary path at point D, the length of CD, is one measure of fiscal space. Alternatively, an adverse surplus shock $(\epsilon_{t+1} < 0)$ would raise debt and reduce the surplus by equal amounts. The reduction in the surplus sends the economy from point C to point

[10] We have assumed that values of the surplus above $\bar{s}$ are not attainable.

[11] Another way to infer solvency at this type of debt-surplus pair is to realize that the country could throw away some of its surplus to move horizontally to the adjustment path BE. If it is solvent when it throws some surplus away, then it must be solvent without throwing away any surplus.

[12] Recall $E_t \psi_{t+1} = E_t \epsilon_{t+1} = E_t \eta_{t+1} = 0$.

[13] Fiscal space is analogous to the distance variable in Daniel and Shiamptanis (2012).

G, while the equal increase in debt moves it vertically from point G to point F. Therefore, another measure of fiscal space is the length of GF. Fiscal space for an expected default or debt shock exceeds fiscal space for a surplus shock ($\epsilon_{t+1}$) because the reduced surplus yields a lower expected present value of future surpluses implying a lower intertemporal-budget-balancing value for debt. Additionally, the shocks could occur in combination, yielding measures of fiscal space between these two distances.

Therefore, when the relevant portion of the boundary path is upward-sloping (BW), fiscal space is not one number but a range of values, captured by the vertical distances between the FD portion of the boundary path and GC. Fiscal space depends on the value of the surplus as well as the value of debt. When the relevant portion of the boundary path is flat (WZ), the two measures of fiscal space are identical.

## 2.6 Fiscal Risk

Fiscal risk is the probability that shocks, together with expectations of default, send the debt-surplus pair beyond the boundary path (in the absence of default). With actual default set to zero, the value of $\eta_{t+1}$ is capital loss on debt due to expected default. When expectations of default are positive, prior to any realization of default, debt is expected to travel above point C due to the risk premium on the interest rate. The higher value of debt implies that the value of exogenous shocks $(\psi_{t+1}, \epsilon_{t+1})$ necessary to send the system above the boundary path is smaller than fiscal space. Using a bounded distribution of shocks, Daniel and Shiamptanis (2012) show that risk of default is increasing at an increasing rate as fiscal space shrinks toward zero, becoming unity along the boundary path. For a debt-surplus pair along the boundary path, the endogenous value of expected future default is high enough that only the best shock could avert default. With the bounded shocks in Daniel and Shiamptanis (2012), the probability of the best shock is zero, implying that the probability of a crisis is unity.

We establish criteria for a country to be almost perfectly safe. Again, assume that the economy is expected to reach point C from an initial position at point A. Let the vertical distance from C to H be equal to an adverse debt shock of $\psi_{t+1}$, created by a three standard deviation shock to $\zeta_{t+1}$. And let the horizontal distance from H to K equal an adverse surplus shock of three standard deviations to $\epsilon_{t+1}$. The adverse surplus shock sends the system from point H diagonally to point J. When the trapezoid labeled CHJK lies fully below the boundary path, virtually no exogenous shocks $(\psi_{t+1}, \epsilon_{t+1})$ could cause default, and endogenous expectations of default are zero. However, when the trapezoid is not fully below, as in the case in Figure 1, some combinations of $\psi_{t+1}$ and $\epsilon_{t+1}$ could cause default. This creates expectations of default, raising the expected value of debt in the absence of default above point C. When some combination of shocks could exhaust fiscal space, expectations of default are positive, implying that actual shocks $\left(\psi_{t+1}, \epsilon_{t+1}\right)$ necessary to create default are smaller than fiscal space due to these expectations.

We use measures of fiscal space to place countries into one of three categories for fiscal risk. We classify the country as "safe", when fiscal space is so large that the trapezoid CHJK lies fully below the boundary path. For intermediate measures of fiscal space, those for which the trapezoid intersects the boundary path, the country is "risky". Finally, when fiscal space is zero with a debt-surplus pair along the boundary path, the country is in the "highest-risk" category. We do not place the probability of a crisis at unity for this case, as in Daniel and Shiamptanis (2012), because our historical estimate of the boundary path could be too low.

# 3 Estimates of Fiscal Feedback Rules

We estimate fiscal feedback rules for ten high-debt countries, Belgium, Canada, France, Greece, Italy, Japan, Portugal, Spain, UK and US using annual data during the period 1970-2007.[14] We cut our sample in 2007 for two reasons. Prior to 2007, countries experienced neither fiscal crises nor very high-debt high-surplus regions,[15] both of which could have caused them to violate their fiscal feedback rules. Second, we want to use data prior to the global financial crisis (in sample data) to assess solvency risks after the global financial crisis between 2008 and 2014 (out of sample).

Prior to estimation we make two empirically-motivated changes to the fiscal feedback rule in equation (17). First, we allow the surplus to depend upon the value of the output gap ($\tilde{y}_t$) and temporary government expenditures ($\tilde{g}_t$) as in Bohn (1998, 2007) and Mendoza and Ostry (2008).[16] We replace $\epsilon_t$ with $\delta_1 \tilde{y}_t + \delta_2 \tilde{g}_t + \tilde{\epsilon}_t$, thereby specifying three different types of shocks to the surplus. The first is an output-gap shock $(\tilde{y}_t)$, which tends to raise the surplus by more than output, thereby increasing $s_t$. The second is a shock to temporary government spending $(\tilde{g}_t)$, and the third $(\tilde{\epsilon}_t)$ aggregates all other shocks including those due to the political process.

The second change to the benchmark fiscal feedback rule is that we account for the possible effect of different values for the growth-adjusted interest rate on the parameters

[14] Details on data are contained in the Data Appendix.

[15] See Section 2.4.

[16] The gap variables are determined by subtracting the trend component, estimated using the Hodrick-Prescott filter, from each observation.

of the fiscal feedback rule. Consider carefully the motivation for this modification.

### 3.1 Interest Rate and the Fiscal Feedback Rule

Equations (21) imply that for countries with positive values for long-run surpluses and debt ($c < 0$), both values are increasing in the growth-adjusted interest rate. If countries want to keep long-run debt below a target value after the interest rate rises, then they could raise the surplus-responsiveness to government debt ($\gamma$), or raise the constant ($c$), reducing the structural deficit. Therefore, we allow the interest rate to affect the parameters of the fiscal feedback rule.

Growth-adjusted interest rates for all countries have similar movements over the sample 1970-2007 with considerable variation. They are negative early, beginning with the 1970's and continuing into the 1980's. The negativity comes from both high inflation and real growth, compared to the nominal interest rate. In the mid-1980's, growth-adjusted interest rates rise with a sharp increase in nominal interest rates, and they remain positive until the late-1990's for some countries and until the end of the sample for others. For all countries except Japan, the interest rates are higher in the middle of the sample than they are early or late. Japan's interest rates rise throughout the sample.

We view these periods in which interest rates took on very different values as different interest rate regimes and test whether the constant ($c$) and responsiveness ($\gamma$) take on different values in different interest rate regimes. To identify different interest rate regimes for each country, we test for multiple break points using the sequential procedure of Bai and Perron (1998, 2003).[17] We allow for up to 5 breaks and serial correlation in the errors. At the 5 percent significance level, we find three separate interest rate regimes for all countries except Italy, which has only two. The dates for each regime over the sample 1970-2007, together with the mean value of the growth-adjusted interest rates in each regime (percents), $r_h$, are given in Table 1.

### 3.2 Empirical Results

The estimating equation, modified to include explicit surplus shocks and different interest rate regimes, is given by

$$s_t = c_h + \beta s_{t-1} + \gamma_h d_{t-1} + \delta_1 \tilde{y}_t + \delta_2 \tilde{g}_t + \tilde{\epsilon}_t, \tag{24}$$

[17] We obtain almost identical results when we use the Bayesian Information Criterion and the modified Schwarz criterion of Liu et al. (1997).

where $h \in (1, 2, 3)$ denotes the interest rate regime, $\gamma_h$ and $c_h$ denote interest-rate-regime specific values for responsiveness and the constant. We use the break dates from Table 1 to construct dummy variables for each interest rate regime, and use them to estimate equation (24).

Results are contained in Table 2. The results imply that surplus responsiveness increases in interest rates, as necessary to mitigate the impact of higher interest rates on long-run values. We find that in the first sub-period, labeled *regime* 1, when interest rates are negative for all countries, responsiveness ($\gamma_1$) is never both positive and significantly different from zero. Therefore, in *regime* 1 with negative interest rates, countries were either responding negatively to debt or were not responding at all. In the middle period, *regime* 2, when interest rates for all countries rise and become positive, responsiveness ($\gamma_2$) rises above zero for all countries and is statistically significant at the 1% percent level for most countries. In *regime* 3 when interest rates fall for all countries except Japan, responsiveness falls ($\gamma_3$) for all countries except Japan. For Japan, the interest rate rises and responsiveness rises. Therefore, for all countries, responsiveness increases systematically with the interest rate.[18] There is no systematic change in the constant across interest rate regimes.

We test whether the interest rate regimes are distinct with F-tests for the equality of the regimes. Table 3 reports the p-values of the F-tests. For almost all the countries, the early low-interest rate regime, *regime* 1, is statistically different from the very highest one. The highest interest rate regime is *regime* 2 for all countries except Japan and is *regime* 3 for Japan. Additionally, for almost all the countries, *regime* 2, which has maximum interest rates for all countries except Japan, is statistically different from *regime* 3.[19]

Next we consider stability in the three interest rate regimes. Stability requires comparison of the responsiveness in the particular interest rate regime ($\gamma_h$) with an interest rate term in that regime from equation (20) ($r_h(1-\beta)$). The interest rates are negative in the first regime for all countries, and the fiscal feedback rules satisfy the stability

[18] As a check on the need to use a fiscal rule with different parameters for different interest rate regimes, we estimated fiscal rules imposing identcal behavior across interest rate regimes. We often failed to find significance for the responsiveness of debt to the surplus, reinforcing our result that the positive and significant responsiveness appears when interest rates are relatively high.

[19] There are two anomalies in Table 3. First, for the UK, the high interest rate regime, *regime* 2, is not statistically different from the later lower interest rate regime, *regime* 3. This might be because the UK experienced the smallest fall in the interest rate between the two regimes. Second, for the US, *regime* 1 is not statistically different from *regime* 2 or from *regime* 3, yet, *regime* 2 is statistically different from *regime* 3. The problem seems to be that estimates in *regime* 1 are imprecise, implying that *regime* 1 is not very different from either *regime* 2 or *regime* 3.

criterion for all countries except France, Greece, Portugal, and the UK.[20] In *regime* 2, the responsiveness is high enough to satisfy the stability requirement for all countries. In *regime* 3, despite the fall in the responsiveness, it remains higher than the interest rate term, satisfying the stability requirement for all countries.

We compute the value of long-run debt for each country in each interest rate regime which satisfies the stability requirement from equation (20). We use the parameter values from Table 2 with the interest rates from Table 1. Long-run values are given in Table 4. For most of the countries, long-run debt is highest in the regime for which the interest rate is highest, implying that countries do not fully adjust the parameters of the fiscal feedback rule to counter the increase in the long-run value of debt implied by a higher interest rate.

# 4 Fiscal Space and Risk Categories

We compute out-of-sample estimates of fiscal space over the period between 2008 and 2014 for each country. The estimates of fiscal space predict the two fiscal crises which occurred and allow us to separate other countries into risk categories. We use estimates of fiscal feedback rules to construct adjustment paths for expected future values of debt and surpluses, conditional on an interest rate regime. To construct the boundary paths, we also need estimates for $\bar{s}$. Therefore, to measure fiscal space, we must first determine the interest rate regime for the post 2007 period and the value of $\bar{s}$.

## 4.1 Interest Rate Regime

The model assumption is that we know the interest rate regime beginning in 2008, not that we are forecasting it. However, it would not be meaningful to assume that we know the regime to be something other than the regime that actually occurred. Therefore, to determine the interest rate regime, we do use post 2007 data.

To determine whether each country was in a low or high interest rate regime after 2007, we repeat the break test now using the full sample (1970-2014). For Belgium, Canada, France, Italy, Japan, UK and the US, we find that there is no change in interest rate regime from *regime* 3 after 2007. This justifies use of the estimated coefficients for the

[20] Failure to satisfy stability criteria when the interest rate is negative should not imply that agents do not lend because governments do not face a binding intertemporal budget constraint. The more they borrow, the more revenue they receive due to the negative interest rate.

*regime* 3 fiscal feedback rule, from Table 2, together with the *regime* 3 interest rates, from Table 1, to estimate expected adjustment paths.

However, for two countries, Canada and the UK, the *regime* 3 interest rate is slightly negative, implying that these countries do not actually face an intertemporal government budget constraint. Our measure of the growth-adjusted interest rate is an ex post realized rate when a government would be using an ex ante expected rate. We do not think it is reasonable that Canada and the UK expected that their budget constraints did not bind over this period. Therefore, for these two countries, we assume that the expected growth-adjusted interest rate was higher than the ex post realized rate and substitute the small positive value for the US growth-adjusted interest rate in *regime* 3.

For Greece, Portugal, and Spain, we find a fourth interest rate regime beginning in 2008, with higher interest rates. We assume a return to the high interest rate *regime* 2, for which we have estimates from Tables 1 and 2.[21]

## 4.2 Estimates of $\bar{s}$ and $\bar{d}$

Tanner (2013) and Collard et al. (2015) choose $\bar{s}$ as the largest surplus the country has ever attained. We follow their lead and choose $\bar{s}$ as the maximum historical surplus in our sample.[22] We also consider the possibility that a country has not actually experienced its surplus limit, $\bar{s}$. If it has experienced its debt limit, then we could use its maximum debt to determine $\bar{s}$ as the surplus peak of the adjustment path whose debt peak is the maximum debt. Finally, it is possible that a particular historical value for the debt-surplus pair implies an even higher $\bar{s}$. We make calculations using the largest historical surplus, the largest historical debt, and the regime-specific debt-surplus pair which provides the largest $\bar{s}$, and select the one which provides the overall largest value for $\bar{s}$. For all countries except Japan, the historical maximum surplus yields the highest boundary path. For Japan, the debt-surplus pair in 2005 provides the highest boundary path.

The estimated fiscal feedback rules (Table 2), together with values for the interest rate (Table 1) and for the surplus limits, allow us to construct the boundary path which peaks at $\bar{s}$ for each country. We use this boundary path to estimate the debt limit, $\bar{d}$,

[21] Interest rates could have risen due to the introduction of risk premia or to the reduction in growth. The measure of the interest rate we need has no risk premium. Therefore, if this were the only possible reason for a break, then we wouldt ignore it. However, the sharp reduction in growth could be responsible for the break. For this reason, we accept the evidence for a break. In the absence of information on *regime 4*, we assume a return to *regime 2*.

[22] We can also justify using historical information to determine ability to pay for the same reasons that private credit markets use a household's history of borrowing and lending to set credit limits.

as the peak debt along the path. Estimates for surplus and debt limits for each country for 2008 are given in Table 5. Countries exhibit large variations in fiscal limits, partially explaining differences in debt tolerance.

Calculations for $\bar{d}$ along the boundary path differ considerably from calculations based on the assumption that a country can maintain its $\bar{s}$ forever. As an example, consider Greece, whose historical maximum surplus in 2008 is 3.06 percent of GDP. Using the *regime* 2 estimated coefficients and interest rate, together with equation (16), the debt limit in 2008 is 108.02 percent of GDP, which is the peak of the debt along the boundary path with peak surplus equal to the historical maximum. In contrast, if we compute the debt limit as the present value of the historical maximum of the surplus, equation (15), we obtain the much larger value of 197.42 percent of GDP. Given the recent financial history of Greece, the second value is unreasonable.

Our measures for the surplus and debt limits can be time-varying. The 2008 measures are based on historical behavior of the debt and the surplus through 2007. However, the actual boundary path could be above the historical boundary path. Going forward from 2008, if the country breaches its historical boundary path in 2008, but retains access to financial markets, we assume that our historical estimate of fiscal limits is too low. We update our estimates for $\bar{s}$ and $\bar{d}$ to lie along the adjustment path for the realized debt-surplus pair, shifting the boundary path upward. With the updated fiscal limits, the country is solvent, but fiscal space is zero placing the country in the highest risk category. The inference of solvency is based on the confidence of lenders that the country can sustain surpluses higher than any historically experienced.

Finally, a country could receive official loans. Since the value of debt in this case is not market determined, the official loan does not have to obey debt limits, implying that the expected present value of the largest future surpluses need not be large enough to equal debt. When the increase in debt is due to an increase in official loans, we do not update our measure of the surplus limit.

## 4.3 Ten High-Debt Developed Countries

We compute measures of fiscal space for each country for each year and place each country into a risk category based on fiscal space. We use each country's risk category to predict the two fiscal crises which occurred and to separate remaining countries into risk categories. Table 6 contains the bounds on fiscal space, the risk category, and the value of debt for each country, either for the crisis year or for the end of our sample, 2014. The high-debt countries we study belong to all three risk categories. The two countries with the highest fiscal space are safe, the three with zero fiscal space are at highest risk, while the others are in the intermediate category with varying risk.

In our model, creditors refuse to lend when the economy is above the boundary locus, and default restores solvency after a crisis. However, when comparing with real-world experience, politics can interfere with market outcomes, separating the timing for loss of market access from that of default. Therefore, we date a fiscal solvency crisis as the date of loss of market access, where private loans are replaced by official loans, even if the actual default comes later (or not yet).[23]

### 4.3.1 Belgium and Canada

Belgium and Canada are two high-debt countries which both belong to the safe category over the entire period following the 2008 global financial crisis. Figure (2) contains the boundary paths for Belgium and Canada, their debt-surplus pairs between 2008 and 2014, and the trapezoids for the position with the highest adjustment path. The trapezoids lie fully below the boundary paths, implying that three standard deviations shocks to both debt and the surplus would not breach the boundary path.

### 4.3.2 Greece and Portugal

Consider the opposite case, Greece and Portugal, which both experienced fiscal crises. Figure (3) contains the boundary paths for both countries for years leading up to their fiscal crises, together with their debt-surplus pairs. For both countries, the 2008 debt-surplus pairs are above the 2008 boundary paths. Since both countries continued to participate in financial markets in 2008 without a fiscal crisis, we update the 2009 boundary paths to be the adjustment path from the 2008 debt-surplus pairs. Debt-surplus pairs on the boundary path, implying zero fiscal space and placing them in the "highest risk" category.

From this position of high risk, adverse shocks in 2009 again pushed both countries above their 2009 boundary paths, implying continuing high risk for both. Since neither country lost access to financial markets, we update the boundary paths again. Greece lost access to financial markets early in 2010, and Portugal in 2011. Two to three years in the highest risk category, predicted both fiscal crises. And debt tolerance for the two countries was very different. Based on Greece's boundary path for the year of the crisis (2010), Greece would have been categorized as risky, not as highest risk, if it had had Portugal's lower pre-crisis debt (Table 6).

[23]Concessionary loans would change the budget constraint. Our model assumes that there are no concessionary loans.

Note the large increases in debt after the crisis for both countries. These values are consistent with our concern that debt might no longer satisfy expected intertemporal budget constraints after loans become official.

### 4.3.3 US and Japan

Figure (4) presents the initial boundary paths for the US and Japan, together with values for the debt-surplus pairs beginning in 2008. Both countries have positive fiscal space in 2008. However, since the trapezoids do not lie fully below the boundary paths, fiscal space is not large enough to eliminate the possibility that shocks in 2009 could send the debt-surplus pairs beyond the historical boundary paths. For both countries, shocks, together with default expectations, did send the 2009 debt-surplus pairs outside their respective boundary paths. In both cases, the 2009 surplus falls by more than the three standard deviations allowed by the trapezoid, implying unusually large adverse surplus shocks were partly responsible for breaching their boundary paths.

Since neither country faced a fiscal crisis in 2009, we update the 2010 boundary paths to be the expected adjustment path from the 2009 debt-surplus pair, placing debt-surplus pairs on their respective boundary paths implying the highest risk category for both countries. In 2010, adverse shocks push the 2010 debt-surplus pair above the 2010 boundary path such that both retained the highest-risk status. This pattern of actual debt-surplus pairs falling outside the historical boundary path continues throughout the period for Japan and until 2013 for the US. Therefore, Japan remains at highest risk until then end of our sample and is an example of the highest debt country also being in the highest risk category. The US experiences positive fiscal space in 2013, but fiscal space remains too small to eliminate risk, placing it in the risky category.

### 4.3.4 France and Spain

Behavior for France and Spain is similar except that these countries exhausted fiscal space later in the sample. Figure (5) presents their initial boundary paths together with values for the debt-surplus pairs between 2008 and 2014. These countries begin the period with large fiscal space which shrinks over time. Spain breached its boundary path in 2013, and France in 2014. In 2013, Spain, facing high interest rates for borrowing, received an official loan, without actually losing access to markets. Spain did access private credit markets in 2014, implying that its intertemporal budget constraint was expected to be satisfied. Even so, fiscal space was zero, placing Spain in the highest risk category. France has not accessed official loans, but exhausted fiscal space in 2014, placing it in the highest risk category. Table 6 shows that both countries had similar debt in 2014. However, the US had more debt, but was in the risky category.

#### 4.3.5 UK and Italy

Finally, consider Figure (6) with boundary paths for the UK and Italy. Over the period, both countries have experienced falling fiscal space, and therefore moving from the safe category to the risky category. Although they have not breached their respective historical boundary paths, fiscal space is not large enough to eliminate the possibility that future shocks could send the debt-surplus pairs beyond the historical boundary paths, as shown by the trapezoids, which do not lie fully below the boundary paths. Although Italy has the second highest debt in the sample (Table 6), fiscal space is positive, placing it in the risky category.

## 5 Conclusion

How do we determine whether or not a country has taken on so much debt that it risks a fiscal solvency crisis? Every country faces limits on the magnitude of the primary surpluses it can raise over time and therefore on the value of the debt it can sustain. And these fiscal limits differ across countries, as evidenced by the Greek crisis at a value of debt/GDP which Belgium successfully managed. Fiscal solvency requires that expected future paths for debt and the surplus remain below the fiscal limits while satisfying the government's expected intertemporal budget constraint. We propose and implement a simple data-based method for estimating fiscal limits and fiscal space, and for separating countries into risk categories.

Our methodology relies on historical measures in two ways. First, we need a way to predict expected future values for the surplus and debt. We estimate a fiscal feedback rule where the surplus responds to its own lag and to lagged debt, but is subject to stochastic shocks. Given a debt-surplus pair, the fiscal feedback rule provides predictions for future surpluses and debt. Second, we need estimates of fiscal limits. We follow Tanner (2013) and Collard et al. (2015) and use the maximum historical value of the surplus as an estimate of the surplus limit. We use the estimated parameters of the fiscal feedback rule to construct the expected future path for the debt and surplus whose peak is the surplus limit. The peak of the debt along this path becomes the debt limit. Surplus and debt limits differ substantially across countries, partly explaining differing tolerance for debt.

To measure fiscal space and assess risk, we use the adjustment path, with peak at the surplus limit, to define a boundary path above which the country is insolvent. For a debt-surplus pair above the boundary path, expected future values of the debt and/or surplus are expected to violate fiscal limits, an impossibility. We define fiscal space as the magnitude of the increase in debt following an adverse shock which would send the debt-surplus pair to the boundary path. Since countries face different fiscal limits, fiscal space is not necessarily lowest for the highest debt countries. We use our measures of fiscal space to separate countries into three risk categories. The highest risk category contains countries which have exhausted their fiscal space. This is uncharted territory – to retain solvency without default, they must be expected to raise surpluses above values deemed feasible by historical experience.

Following the 2008 global financial crisis, both Greece and Portugal exhausted their fiscal space, placing them in the highest risk category. Several years in the highest risk category predicted the fiscal crises they experienced. All other countries experienced shrinking fiscal space after 2008, with adverse consequences for risk for all countries, except Belgium and Canada, which remained in the safe category. The US and Japan exhausted fiscal space one year after the global financial crisis, with the US moving back to positive territory only in 2013 and Japan never regaining positive fiscal space. France and Spain experienced falling fiscal space, with Spain receiving an official loan in 2013 and France exhausting fiscal space in 2014. Italy and the UK have experienced falling fiscal space and therefore rising risk, but have not exhausted their fiscal space. Our sample ends with France, Spain, and Japan in the highest risk category, with Japan having maintained that status since 2008. Continued solvency for these countries rests on confidence from lenders that they can attain surpluses higher than historically observed, warning of potential future fiscal crises.

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# 6 Data Appendix

| Variable | Description | Source |
|---|---|---|
| $s_t$ | primary surplus as a percentage of GDP | OECD (Economic Outlook No. 96) |
| $d_t$ | gross government debt as a percentage of GDP | OECD (Economic Outlook No. 96) and AMECO database |
| $\tilde{y}_t$ | cyclical component of the log real GDP obtained from the Hodrick-Prescott filter | OECD (Economic Outlook No. 96) and Authors' calculations |
| $\tilde{g}_t$ | cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter | OECD (Economic Outlook No. 96) and Authors' calculations |
| $\rho_t$ | nominal GDP growth rate | OECD (Economic Outlook No. 96) |
| $i_t$ | long-run nominal interest rate on government bonds | OECD (Economic Outlook No. 96) |
| $r_t$ | growth-adjusted interest rate is as $r_t = \frac{1+i_{t-1}}{1+\rho_t} - 1$ | Authors' calculations |

The sample is from 1970-2014. We use 1970-2007 to estimate the fiscal feedback rules (in sample estimation), and we use 2008-2014 for the assessment of risk (out of sample).

**Summary Statistics (1970-2007)**

| | Debt as % of GDP | | | Surplus as % of GDP | | |
|---|---|---|---|---|---|---|
| | Min | Mean | Max | Min | Mean | Max |
| Belgium | 54.28 | 98.31 | 134.65 | -9.37 | 0.59 | 6.10 |
| Canada | 44.08 | 71.45 | 109.40 | -6.35 | -0.61 | 5.95 |
| France | 20.39 | 40.39 | 67.20 | -3.75 | -0.72 | 1.10 |
| Greece | 14.98 | 59.39 | 106.85 | -3.82 | -0.67 | 3.06 |
| Italy | 35.71 | 82.27 | 117.17 | -7.74 | -1.22 | 5.65 |
| Japan | 69.95 | 79.78 | 169.55 | -9.07 | -2.50 | 3.22 |
| Portugal | 13.30 | 45.66 | 69.17 | -4.75 | -1.18 | 2.23 |
| Spain | 11.50 | 36.51 | 65.57 | -7.60 | -1.89 | 3.48 |
| UK | 31.36 | 48.20 | 75.79 | -5.06 | -0.10 | 7.83 |
| US | 42.63 | 55.85 | 71.20 | -4.19 | -0.65 | 3.59 |

# 7 Tables

**Table 1 Interest Rate Regimes**

| | *regime* 1 | | *regime* 2 | | *regime* 3 | |
|---|---|---|---|---|---|---|
| | Period | $r_1$ | Period | $r_2$ | Period | $r_3$ |
| Belgium | 1970-1978 | -2.90 | 1979-1996 | 3.72 | 1997-2007 | 0.19 |
| Canada | 1970-1981 | -3.12 | 1982-1997 | 3.59 | 1998-2007 | -0.68 |
| France | 1970-1989 | -1.01 | 1990-1997 | 4.18 | 1998-2007 | 0.29 |
| Greece | 1970-1985 | -6.65 | 1986-2000 | 1.55 | 2001-2007 | -3.93 |
| Italy | 1970-1980 | -6.58 | 1981-2007 | 1.93 | | |
| Japan | 1970-1974 | -7.97 | 1975-1997 | 0.23 | 1998-2007 | 1.69 |
| Portugal | 1970-1980 | -7.27 | 1981-1996 | 2.65 | 1997-2007 | -1.02 |
| Spain | 1970-1979 | -6.71 | 1980-1997 | 2.10 | 1998-2007 | -2.92 |
| UK | 1970-1980 | -3.62 | 1981-1995 | 2.10 | 1996-2007 | -0.11 |
| US | 1970-1979 | -2.21 | 1980-1991 | 2.64 | 1992-2007 | 0.02 |

**Table 2 Fiscal Rules with Interest Rate Dependency**

| | Belgium | Canada | France | Greece | Italy | Japan | Portugal | Spain | UK | US |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_1$ | -2.534 | -6.346* | 0.499 | 1.672‡ | -5.241† | -1.807 | 0.637 | -4.982† | 6.233† | -11.022* |
| | (6.601) | (3.305) | (0.697) | (0.382) | (1.960) | (5.019) | (0.799) | (2.535) | (2.555) | (5.981) |
| $c_2$ | -10.907‡ | -6.164‡ | -3.744‡ | -5.05‡ | -7.711‡ | -2.275† | -4.434* | -4.781‡ | -4.331‡ | -8.224‡ |
| | (3.767) | (0.947) | (0.517) | (1.700) | (1.837) | (1.086) | (2.633) | (1.272) | (1.464) | (2.460) |
| $c_3$ | -4.961* | -3.885* | -4.346 | -6.416 | | -12.038‡ | -2.756 | -1.098 | -3.136 | -5.248† |
| | (2.663) | (2.153) | (4.067) | (7.032) | | (3.563) | (2.228) | (1.096) | (3.508) | (2.120) |
| $\beta$ | 0.353† | 0.588‡ | 0.290† | 0.368‡ | 0.449‡ | 0.619‡ | 0.253* | 0.442‡ | 0.614‡ | 0.620‡ |
| | (0.136) | (0.069) | (0.138) | (0.122) | (0.130) | (0.131) | (0.142) | (0.141) | (0.140) | (0.096) |
| $\gamma_1$ | 0.011 | 0.113 | -0.037* | -0.110‡ | 0.055 | 0.041 | -0.118* | 0.231 | -0.104† | 0.211 |
| | (0.119) | (0.067) | (0.021) | (0.019) | (0.039) | (0.380) | (0.069) | (0.184) | (0.042) | (0.128) |
| $\gamma_2$ | 0.101‡ | 0.077‡ | 0.075‡ | 0.067‡ | 0.084‡ | 0.029* | 0.085* | 0.077‡ | 0.097‡ | 0.160‡ |
| | (0.033) | (0.011) | (0.010) | (0.020) | (0.020) | (0.016) | (0.052) | (0.019) | (0.032) | (0.046) |
| $\gamma_3$ | 0.070† | 0.051* | 0.055 | 0.057 | | 0.070‡ | 0.025 | 0.035 | 0.077 | 0.076† |
| | (0.028) | (0.026) | (0.064) | (0.070) | | (0.024) | (0.039) | (0.020) | (0.082) | (0.034) |
| $\delta_1$ | 0.336† | 0.346‡ | 0.334‡ | 0.013 | 0.164 | 0.002 | 0.089 | 0.288‡ | 0.137† | 0.305‡ |
| | (0.148) | (0.063) | (0.086) | (0.068) | (0.211) | (0.120) | (0.047) | (0.074) | (0.066) | (0.060) |
| $\delta_2$ | -0.223 | -0.396‡ | -0.559‡ | -0.132‡ | -0.153 | -0.241 | -0.002 | -0.138* | -0.519‡ | -0.657‡ |
| | (0.227) | (0.071) | (0.147) | (0.073) | (0.124) | (0.229) | (0.083) | (0.082) | (0.199) | (0.119) |
| $\bar{R}^2$ | 0.907 | 0.960 | 0.683 | 0.761 | 0.904 | 0.760 | 0.405 | 0.929 | 0.712 | 0.846 |
| $\sigma_{\bar{\epsilon}}$ | 1.264 | 0.711 | 0.620 | 0.944 | 1.132 | 1.495 | 1.402 | 0.802 | 1.318 | 0.714 |

Note: The ∗, † and ‡ indicate statistical significance at the 10, 5 and 1 percent level, respectively.

**Table 3 F-tests on Regime Equality**

| | Belgium | Canada | France | Greece | Italy | Japan | Portugal | Spain | UK | US |
|---|---|---|---|---|---|---|---|---|---|---|
| p($regime1$=$regime2$) | 0.089 | 0.000 | 0.000 | 0.000 | 0.030 | 0.899 | 0.054 | 0.099 | 0.006 | 0.592 |
| p($regime2$=$regime3$) | 0.007 | 0.104 | 0.000 | 0.000 | - | 0.004 | 0.089 | 0.013 | 0.668 | 0.000 |
| p($regime1$=$regime3$) | 0.508 | 0.027 | 0.386 | 0.000 | - | 0.021 | 0.175 | 0.008 | 0.079 | 0.385 |

**Table 4 Long Run Debt (% of GDP)**

| | Belgium | Canada | France | Greece | Italy | Japan | Portugal | Spain | UK | US |
|---|---|---|---|---|---|---|---|---|---|---|
| $regime1$ | 86.49 | 50.61 | (unstable) | (unstable) | 57.57 | 25.38 | (unstable) | 18.53 | (unstable) | 50.23 |
| $regime2$ | 141.53 | 98.40 | 83.36 | 89.19 | 105.35 | 79.75 | 67.27 | 73.15 | 48.59 | 54.95 |
| $regime3$ | 71.99 | 72.74 | 81.50 | 78.72 | - | 188.75 | 84.80 | 21.03 | 40.58 | 69.07 |

**Table 5 Fiscal limits (% of GDP)**

| | Belgium | Canada | France | Greece | Italy | Japan | Portugal | Spain | UK | US |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{s}_{2008}$ | 6.10 | 5.95 | 1.10 | 3.06 | 5.65 | 3.38 | 2.23 | 3.48 | 7.83 | 3.59 |
| $\bar{d}_{2008}$ | 137.97 | 142.04 | 94.06 | 108.02 | 135.42 | 190.18 | 71.74 | 90.60 | 99.16 | 96.04 |

**Table 6 Fiscal Space (% of GDP)**

| | Belgium | Canada | France | Italy | Japan | Spain | UK | US | Greece | Portugal |
|---|---|---|---|---|---|---|---|---|---|---|
| Fiscal space | end of the sample | | | | | | | | year of crisis | |
| largest | 33.79 | 48.02 | 0 | 4.82 | 0 | 0 | 11.71 | 3.10 | 0 | 0 |
| smallest | 28.46 | 27.68 | 0 | 4.82 | 0 | 0 | 10.70 | 3.10 | 0 | 0 |
| Risk category | safe | safe | highest | risky | highest | highest | risky | risky | highest | highest |
| Debt level | 106.47 | 93.86 | 95.02 | 132.11 | 229.99 | 97.67 | 89.36 | 109.71 | 130.7 | 97.6 |

Note: Greece (Portugal) received its first bailout in May 2010 (May 2011) after losing access to private credit markets. We list debt at the end of the year prior to each crisis 2009 (2010).

Figure 1: Fiscal space

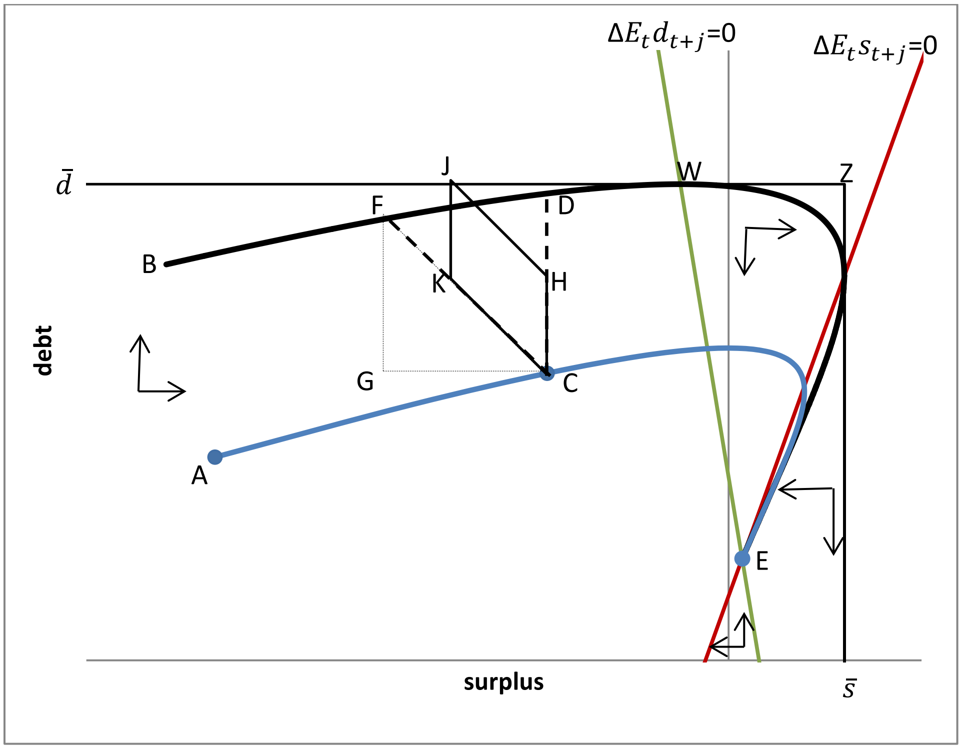


Figure 2: Belgium and Canada

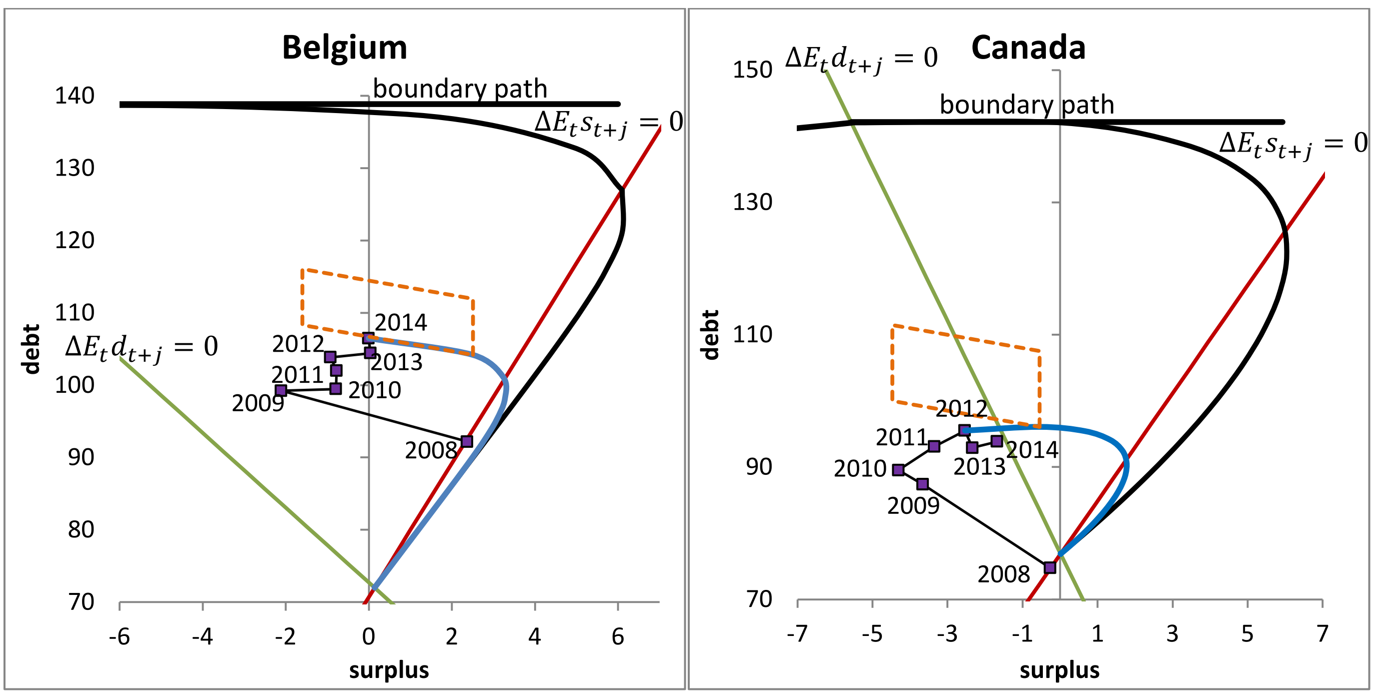

Figure 3: Greece and Portugal

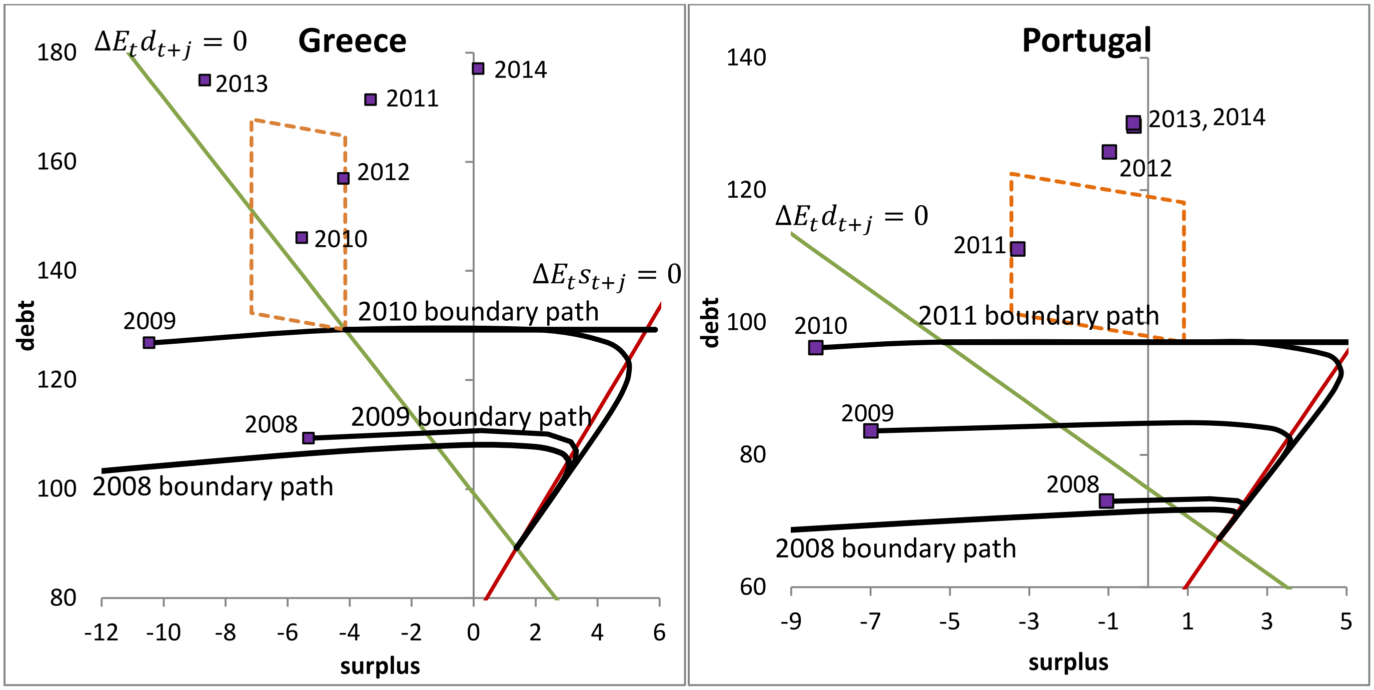


Figure 4: US and Japan

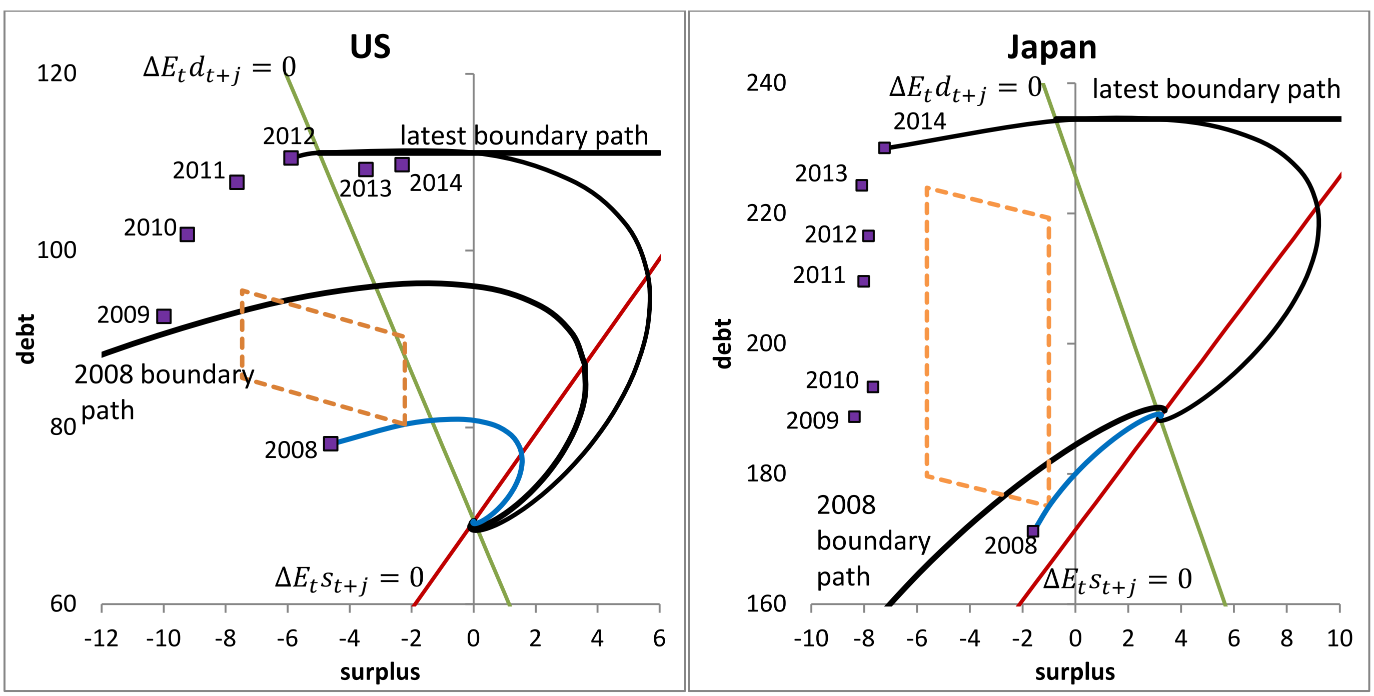

Figure 5: France and Spain

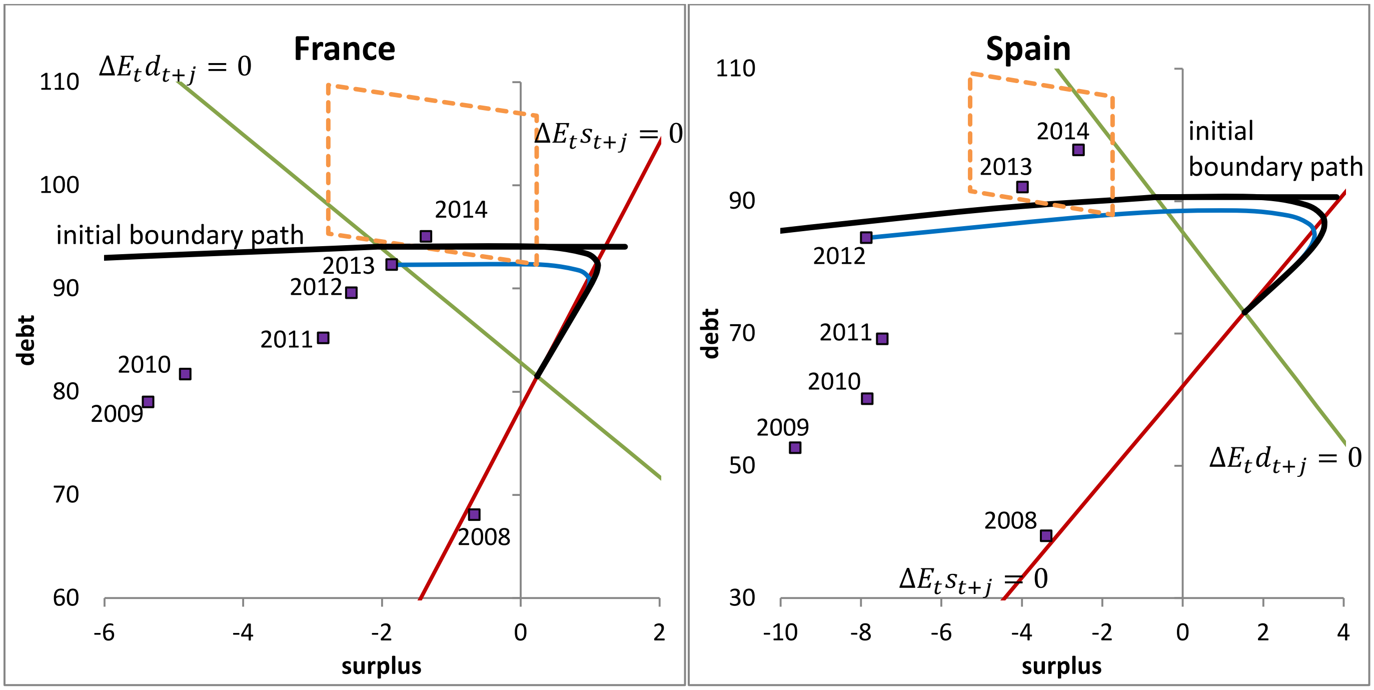


Figure 6: Italy and Spain