A Quantitative Assessment of Marriage Markets: How Inequality is Remaking the American Family

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- *In the US, marriage rates for all groups have declined significantly since the seventies.*
- Following Goldin and Katz, Cabonne and Cahn argue that the availability of the pill enabled single women to pursue higher education and a career without having to forgo premarital sex. This led to an increase in female educational attainment, with more women than men attending and graduating from college by the nineties.
- Female college graduates' focus on their careers differentiated them from high school graduates as potential spouses and made them attractive to increasingly scarce male college graduates.
- At the bottom of the earnings distribution, male high school dropouts were increasingly detached from the labor market, rendering them unmarriageable from the perspective of potential mates, primarily female high school dropouts. However, these women did not stop having children and the fraction of single parent headed households, and the fraction of single parent headed households, which tend to have low income, grew.
- So, more positive assortative matching (PAM) by educational attainment at the top with dual spousal earnings, and a disproportionate retreat from marriage at the bottom of the educational distribution led to increased family earnings inequality. The problem is exacerbated by the decline in manufacturing which shifted a significant mass of men from middle class to working class.
- *CC and other observers argue that the recent increase in earnings inequality and the increase in PAM by educational attainment caused recent declines in marriage rates.*
- *How quantitatively important are these hypotheses in explaining recent changes in marital patterns?*
- We will use a difference in differences approach. Consider each state year, (s, t), a separate marriage market.
- Holding age constant, let the differences between individuals of the same gender be their schooling attainment and race, black vs white.

7 Data

US census 1970 to 2012. We use the ACS from 2010-2012 for 2012.

Each state, year, race (black and white) is a separate marriage market.

We consider women between 26-30 and men between 27-31.

We define a type of individual by their educational attainment: Less than high school, High school graduate and some college, University graduate.

So there are potentially 9 types of marital matches in each marriage market.

Average wage of a type is the average annual earnings of a full time worker by education, state, year and race.

Tables & Figures

Table 1: Marriage rates for young women, 1970 and 2012.

	1970	2012
Total	75.3%	37.3%
Less than high school	72.6%	33.9%
High school or some college	77.8%	34.5%
College	69.1%	42.6%

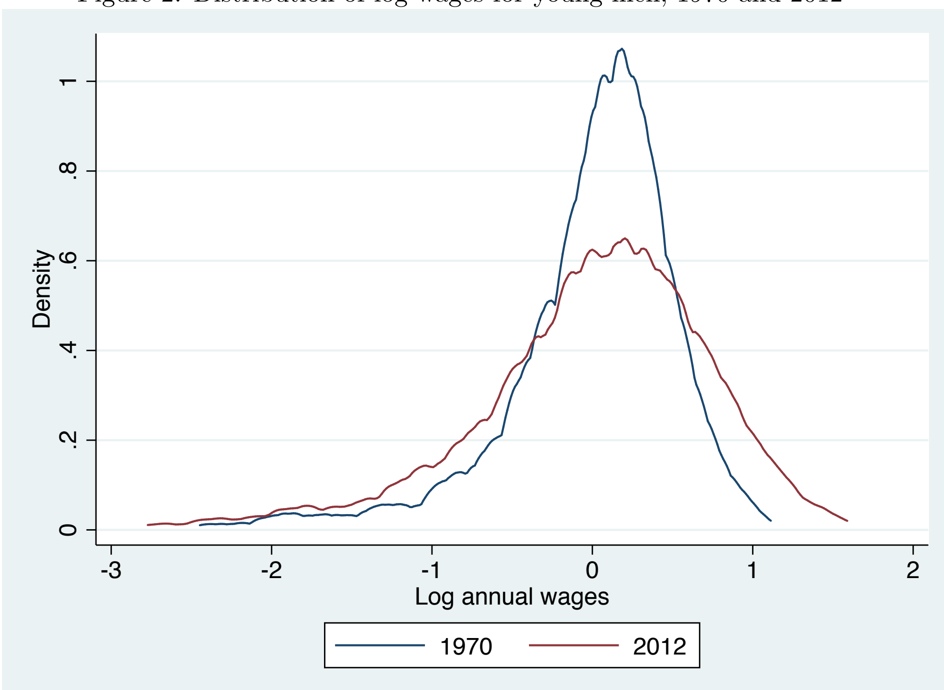
Data is from the 1970 Census and the 2010-2012 American Community Surveys. The sample construction is described in the text; further details are available upon request.

Table 2: Education levels by sex and year

	Male		Female		Sex ratio (M/F)	
	1970	2012	1970	2012	1970	2012
Less than high school	28.4%	10.8%	26.1%	7.3%	1.028	1.483
High school or some college	52.0%	61.8%	61.2%	57.8%	0.803	1.080
College	19.6%	27.4%	12.7%	34.9%	1.460	0.792

Data comes from the 1970 Census and the 2010-2012 American Community Suvey. The sample construction is described in the text; further details are available upon request.

Figure 2: Distribution of log wages for young men, 1970 and 2012

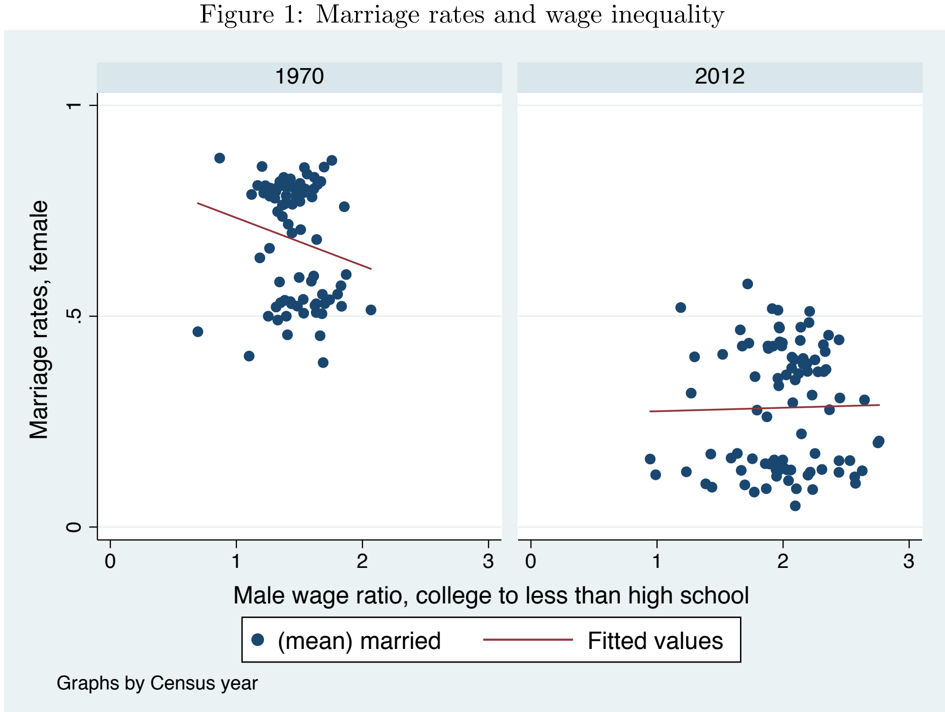


Data comes from the 1970 Census and the 2010-2012 American Community Suvey. The graph depicts the kernel density of log annual wages for young males who work full-time and full year. The sample construction is described in the text; further details are available upon request

1 Summary

- The effects of changes in educational attainment and earnings inequality on marital outcomes are qualitatively consistent with CC hypotheses.
- The quantitive effects on marital outcomes due to these changes are too small to explain the large declines in observed marriage rates.
- So the large recent declines in marriage rates remain largely unexplained.

Figure 1: Marriage rates and wage inequality



This graph plots the state by race-level ratio of wages for male college educated workers to workers with less than high school, against marriage rates. Data comes from the 1970 Census and the 2010-2012 American Community Suvey. Further details of the sample construction are available upon request.

2 Marriage matching function

- There are I types of men and J types of women.
- M vector with element m_i. F vector with element f_j. Π vector of parameters where there are not more than IJ parameters.
- A marriage matching function is an $I \times J$ matrix $\mu(M, F; \Pi)$ whose i, j element is μ_{ij}:

$$\mu_{0j} + \sum_{i=1}^{I} \mu_{ij} = f_j \ \forall \ j \tag{1}$$

$$\mu_{i0} + \sum_{j=1}^{J} \mu_{ij} = m_i \ \forall \ i \tag{2}$$

$$\mu_{ij} \geq 0 \ \forall \ i, j \tag{3}$$

- Given $\mu(.)$, we can study how changes in M and F, and how changes in earnings inequality affect Π which in turn, affect marital matching.

3 A behavioral approach to MMF

- We propose a transferable utility model of the marriage market.
- There are three conceptual benefits for considering transferable utility models of the marriage market.
  1. Marriage market equilibrium must satisfy all the accounting constraints.
  2. Reduced form for equilibrium quantities of a market clearing model do not include equilibrium transfers.
  3. We do not impose apriori ordering of spousal preferences.
- Marital output of an i, j pair depends on i and j.
- $I \times J$ marital outputs plus $I + J$ outputs of types being single.
- Transferable utility models maximize the sum of marital output in the society. See Galichon and Salanie.
- McFadden's (1974) extreme value random utility functions for choices over spouses.
- CS marriage matching function:

$$\frac{\mu_{ij}}{\sqrt{(m_i - \sum_k \mu_{ik})(f_j - \sum_l \mu_{lj})}} = \Pi_{ij} \ \forall (i, j)$$

- MMF will fit any observed marriage distribution.
- Given M, F, and Π, the MMF generates a unique μ.
- Changes in educational attaiment changes M and F, and changes in earnings inequality changes Π. So we can study changes in μ.

4 The CS model

4.1 Quasi demand for wives

- Let the utility of male g of type i who marries a female of type j be:

$$V_{ijg} = \widetilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg}, \qquad \text{where} \tag{4}$$

$\widetilde{\alpha}_{ij}$: Systematic gross return to male of type i married to female of type j.
τ_{ij}: Equilibrium transfer made by male of type i to spouse of type j.
ε_{ijg} : i.i.d. random variable with type I extreme value distribution.

- The payoff to g from remaining unmarried, denoted by $j = 0$, is:

$$V_{i0g} = \widetilde{\alpha}_{i0} + \varepsilon_{i0g} \tag{5}$$

  where ε_{i0g} is also an i.i.d. random variable with type I extreme value distribution.

Individual g will choose according to:

$$V_{ig} = \max_j [V_{i0g}, .., V_{ijg}, .., V_{iJg}] \tag{6}$$

- When there are lots of type i men, McFadden shows that the

quasi-demand function for j type spouse:

$$\ln \frac{\mu^d_{ij}}{\mu^d_{i0}} = \alpha_{ij} - \alpha_{i0} - \tau_{ij}$$

4.2 Quasi supply of wives

- Let the utility of female k of type j who marries a male of type i be:

$$U_{ijk} = \gamma_{ij} + \tau_{ij} + e_{ijk}, \quad \text{where} \tag{7}$$

γ_{ij}: Systematic gross return to female of type j married to male of type i.
e_{ijk} : i.i.d. random variable with type I extreme value distribution.

- The payoff to k from remaining unmarried, denoted by $i = 0$, is:

$$U_{0jk} = \gamma_{0j} + e_{0jk} \tag{8}$$

  where e_{0jk} is also an i.i.d. random variable with type I extreme value distribution.

Woman k will choose according to:

$$U_{ik} = \max_i [U_{0jk}, .., V_{ijk}, .., V_{Ijk}] \tag{9}$$

When there are lots of type j women, quasi supply function of i type spouse:

$$\ln \frac{\mu_{ij}^s}{\mu_{0j}^s} = \gamma_{ij} - \gamma_{0j} + \tau_{ij}$$

5 Market clearing

- The marriage market clears when given equilibrium transfers τ_{ij},

$$\mu_{ij} = \mu_{ij}^d = \mu_{ij}^s \ \forall i, j$$

- Marriage matching function

$$\ln \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} = \frac{\alpha_{ij} - \alpha_{i0} + \gamma_{ij} - \gamma_{0j}}{2} = \pi_{ij} \ \forall i, j \tag{10}$$

- Note that the LHS of (10) is observed. So π_{ij} is identified. What about α_{ij}, α_{i0}, γ_{ij}, γ_{0j}?

- Decker, et. al. shows that for all admissible parameters, a unique equilibrium exists. What this means is that π_{ij} is an alternative description of the marriage market. In a single cross section, the CS MMF will fit the data exactly.

- Decker, et. al. also derive some comparative statics. E.g. a type i male marriage rate is weakly decreasing in type l males. A type j female marriage rate is weakly increasing in type l males.

6 Positive assortative matching

- Let the heterogeneity across males (females) be one dimensional and ordered. Without loss of generality, let male (female) ability be increasing in i (j).
- Then using (10), the local log odds for (i,j) is:

$$\begin{aligned} l(i,j) &= \ln \frac{\mu_{ij}\mu_{i+1,j+1}}{\mu_{i+1,j}\mu_{i,j+1}} = \pi_{ij} + \pi_{i+1,j+1} - \pi_{i+1,j} - \pi_{i,j+1} & (11) \\ &= \alpha_{ij} + \gamma_{ij} + \alpha_{i+1,j+1} + \gamma_{i+1,j+1} - (\alpha_{i+1,j} + \gamma_{i+1,j}) - (\alpha_{i,j+1} + \gamma_{i,j+1}) & (12) \end{aligned}$$

- Only $\alpha_{ij} + \gamma_{ij}$, marital surplus for the married couple appears in (12) and not α_{i0} or γ_{0j}.
- When $\alpha_{ij} + \gamma_{ij} > 0$ for all (i,j), then marital surplus is supermodular in (i,j). $l(i,j) > 0$ for all (i,j) which statisticians say is positive assortative matching by (i,j).
- When $\alpha_{ij} + \gamma_{ij} < 0$ for all (i,j), then marital surplus is submodular in (i,j). $l(i,j) < 0$ for all (i,j) which statisticians say is negative assortative matching by (i,j). **Becker says that marital surplus is submodular in spousal wages.**

- Let a marriage market be denoted by s and t.
- Let the average wages of individuals of type i and j in market st be w_i^{st} and w_j^{st} respectively.
- Let

$$\begin{aligned}\alpha_{ij}^{st} + \gamma_{ij}^{st} &= \lambda_{ij}^{t} + \lambda_s + (\ln w_i^{st})\lambda_1 + (\ln w_j^{st})\lambda_2 \\ &+ (\ln w_i^{st})(\ln w_j^{st})\lambda_3 + \varepsilon_{ij}^{st} \\ \alpha_{i0}^{st} &= \rho_i^t + \rho_s + \ln w_i^{st}\rho_1 + \varepsilon_{i0}^{st} \\ \gamma_{0j}^{st} &= \beta_j^t + \beta_s + \ln w_j^{st}\beta_2 + \varepsilon_{0j}^{st}\end{aligned}$$

  λ_3 is the complimentary parameter of spousal wages. Becker says $\lambda_3 < 0$.
- Let

$$\begin{aligned}\alpha_{ij}^{st} + \gamma_{ij}^{st} &= \lambda_{ij}^{t} + \lambda_s + (\ln w_i^{st})\lambda_1 + (\ln w_j^{st})\lambda_2 \\ &+ (\ln w_i^{st})(\ln w_j^{st})\lambda_3 + \varepsilon_{ij}^{st} \\ \alpha_{i0}^{st} &= \rho_i^t + \rho_s + \ln w_i^{st}\rho_1 + \varepsilon_{i0}^{st} \\ \gamma_{0j}^{st} &= \beta_j^t + \beta_s + \ln w_j^{st}\beta_2 + \varepsilon_{0j}^{st}\end{aligned}$$

- Then

$$\begin{aligned}\ln \frac{\mu_{ij}^{st}}{\sqrt{\mu_{i0}^{st}\mu_{0j}^{st}}} &= \pi_{ij}^{st} = \lambda_{ij}^{t} + \lambda_s + (\ln w_i^{st})\lambda_1 + (\ln w_j^{st})\lambda_2 + (\ln w_i^{st})(\ln w_j^{st})\lambda_3 \\ &- (\rho_i^t + \rho_s + \ln w_i^{st}\rho_1 + \varepsilon_{i0}^{st}) - (\beta_j^t + \beta_s + \ln w_j^{st}\beta_2 + \varepsilon_{0j}^{st}) + \varepsilon_{ij}^{st} \\ &= \pi_{ij}^{t} + \pi_s + (\ln w_i^{st})\pi_1 + (\ln w_j^{st})\pi_2 + (\ln w_i^{st})(\ln w_j^{st})\pi_3 + \epsilon_{ij}^{st}\end{aligned}$$

- π_3 identifies λ_3 which is the complimentary parameter of spousal wages.

$$l(i,j,s,t) = \ln \frac{\mu_{ij}^{st}\mu_{i+1,j+1}^{st}}{\mu_{i+1,j}^{st}\mu_{i,j+1}^{st}} = \pi_{ij}^{st} + \pi_{i+1,j+1}^{st} - \pi_{i+1,j}^{st} - \pi_{i,j+1}^{st} \tag{13}$$

$$= \lambda_{ij}^{t} + \lambda_{i+1,j+1}^{t} - \lambda_{i+1,j}^{t} - \lambda_{i,j+1}^{t} + \varepsilon_{ij}^{st} + \varepsilon_{i+1,j+1}^{st} - \varepsilon_{i+1,j}^{st} - \varepsilon_{i,j+1}^{st} \tag{14}$$

$$+ \lambda_3(\ln w_i^{st} - \ln w_{i+1}^{st})(\ln w_j^{st} - \ln w_{j+1}^{st}) \tag{15}$$

How can you model an increase in earnings inequality? What effect does this have on PAM?

Table 3: Male and female wages, by education group and year

	1970		
	Female wage	Male wage	Ratio of female to male
Less than high school	\$12,032.77	\$31,304.59	0.38
High school or some college	\$16,728.83	\$38,492.95	0.43
College	\$23,658.16	\$46,503.83	0.51
	2012		
	Female wage	Male wage	Ratio of female to male
Less than high school	\$11,622.98	\$17,896.95	0.65
High school or some college	\$17,797.99	\$25,954.16	0.69
College	\$29,053.78	\$39,621.95	0.73

This table shows average annual wages for full-time, full-year workers. Data comes from the 1970 Census and the 2010-2012 American Community Suvey. The sample construction is described in the text; further details are available upon request.

Table 4: Estimation of marriage matching model

	Dependent variable: $ln\left(\frac{\mu_{ijrst}}{\sqrt{\mu_{i0rst}\mu_{0jrst}}}\right)$			
	Linear	Linear	Log	Log
Male wage	$-5.79*10^{-6*}$	$9.42*10^{-6*}$	-0.112	6.243***
	$(3.09*10^{-6})$	$(5.01*10^{-6})$	(0.111)	(1.318)
Female wage	$7.12*10^{-6}$	$2.67*10^{-5***}$	0.067	6.600***
	$(5.37*10^{-6})$	$(8.14*10^{-6})$	(0.134)	(1.355)
Male x female wage interaction		$-5.77*10^{-10***}$		-0.636***
		$(1.48*10^{-10})$		(0.132)
R^2	0.906	0.906	0.906	0.907
Observations	3,688	3,688	3,688	3,688

Data come from the 1970-2000 Public Use Census samples and the 2010-2012 American Community Surveys. The sample construction is described in the text; further details are available upon request. Regressions are weighted by the number of marriages in each cell.

Table 5: Marriage rates for women: actual and simulated with linear model

	Actual		Simulations		
	1970	2012	(A)	(B)	(C)
Total	75.3%	37.3%	71.9%	72.8%	36.2%
By education level:					
No high school	72.6%	33.9%	83.6%	82.8%	35.1%
High school or some college	77.8%	34.5 %	78.5%	79.4%	33.6%
College degree	69.1%	42.6 %	58.8%	60.1%	40.9%
Supplies			2012	2012	2012
Match values			1970	1970	2012
Wages			1970	2012	1970
Model			Logs with interaction	Logs with interaction	Logs with interaction

Data come from the 1970-2000 Public Use Census samples and the 2010-2012 American Community Surveys. The sample construction is described in the text; further details are available upon request.

Table 6: Local log odds

	1970	2012	Change
No high school-high school	1.674	2.657	0.983
High school-college	2.450	2.277	-0.173

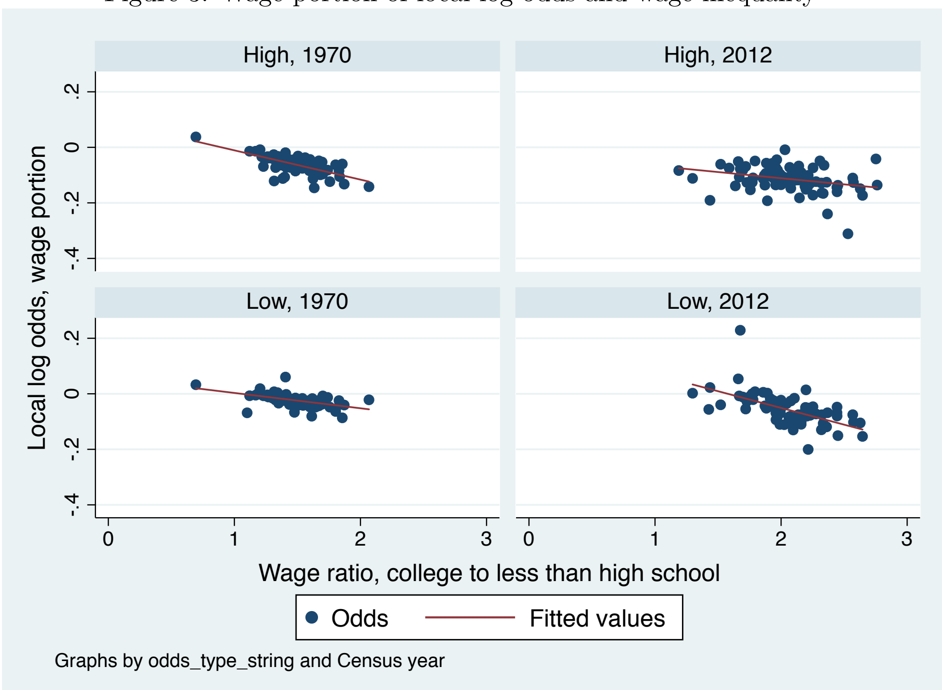
Data come from the 1970 Public Use Census sample and the 2010-2012 American Community Surveys. The sample construction is described in the text; further details are available upon request. Wives' and husbands' education level is measured in three categories: less than high school, high school/some college, and college degree or higher.

Table 7: Local log odds: actual and simulated

	Actual, 1970	Actual, 2012	Simulations		
			(A)	(B)	(C)
HH-MM	2.450	2.277	2.472	2.388	2.373
MM-LL	1.675	2.657	1.708	1.669	2.729
Supplies			2012	2012	2012
Match values			1970	1970	2012
Wages			1970	2012	1970
Model			Logs with interaction	Logs with interaction	Logs with interaction

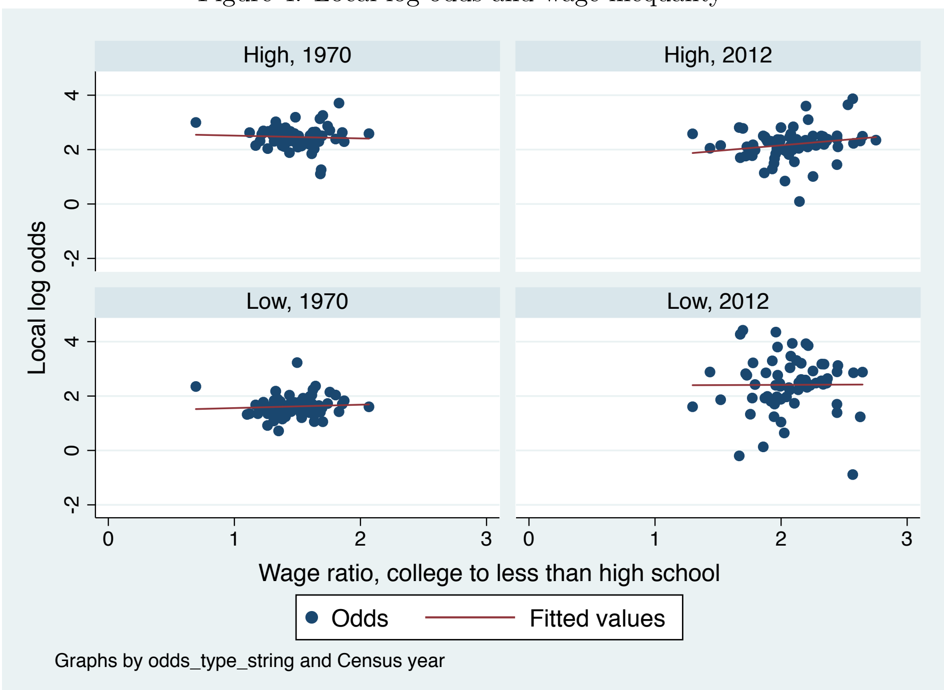
Data come from the 1970-2000 Public Use Census samples and the 2010-2012 American Community Surveys. The sample construction is described in the text; further details are available upon request.

Figure 3: Wage portion of local log odds and wage inequality



This graph plots the state by race-level ratio of wages for college educated workers to workers with less than high school, against wage portion of local log odds for college to high school/some college (high), and high school/some college to less than high school (low) . Data comes from the 1970 Census and the 2010-2012 American Community Suvey. The sample construction is described in the text; further details are available upon request.

Figure 4: Local log odds and wage inequality



This graph plots the state-level ratio of wages for college educated workers to workers with less than high school, against local log odds for college to high school/some college (high), and high school/some college to less than high school (low) . Data comes from the 1970 Census and the 2010-2012 American Community Suvey. The sample construction is described in the text; further details are available upon request.

8 References

Choo, Eugene and Aloysius Siow. 2006. "Who Marries Whom and Why." *Journal of Political Economy* 114 (1): 175-201.
Cornelson, Kirsten and Aloysius Siow. forthcoming. "A Quantitative Assessment of *Marriage Markets: How Inequality is Remaking the American Family.*" *Journal of Economic Literature.*
Decker, Colin, Elliott H. Lieb, Robert J. McCann, and Benjamin K. Stephens. "Unique equilibria and substitution effects in a stochastic model of the marriage market." *Journal of Economic Theory* 148, no. 2 (2013): 778-792.
Galichon, Alfred and Salanié, Bernard, Cupid's Invisible Hand: Social Surplus and Identification in Matching Models (September 17, 2012). SSRN: http://ssrn.com/abstract=1804623
Siow, Aloysius. April 2015. "Testing Becker's Theory of Positive Assortative Matching." *Journal of Labor Economics.*