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Tax Austerity: Does it Avert Solvency Crises?

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Abstract

Many high-debt countries are adopting tax austerity, whereby governments raise their tax rate as the debt level rises with the hope to dispel future solvency crisis. This paper investigates the implications of tax austerity on the likelihood of a solvency crisis. A solvency crisis occurs once adverse shocks push the debt level above its effective debt limit, which is the maximum level of debt that the government can repay. We derive the effective debt limit and show that its position depends on tax austerity. We find that high-debt countries like Italy that undergo tax austerity could lower their effective debt limit and induce a solvency crisis in the near future.

- *Key Words:* Debt limit; Fiscal limit; Austerity; Solvency Crisis; Default
- *JEL Classification:* E62; F34; H30; H60

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1 Introduction

The European debt crisis sparked an era of austerity, whereby governments aggressively raised taxes. Many high-debt countries were pressured to adopt austerity measures to rein in their elevated debt levels with the hope to allay the possibility of a future solvency crisis. But, do austerity measures avert the likelihood of future insolvency? In Greece, it took a decade for the government to finally escape the crisis, despite the strict austerity demanded by international creditors. In Italy, the government resisted additional austerity and instead eased up on austerity, despite its ever-growing debt level. In Portugal, the government abandoned the austerity measures imposed by its creditors, defying critics who insist on austerity as the answer to a solvency crisis. The EU officials seem to believe that austerity measures avert solvency crisis, whereas some European countries appear to doubt whether austerity will ever end a crisis or prevent one. Here, we evaluate the efficacy of austerity.

This paper focuses on tax austerity. We use a small open economy model in which the government increases the tax rate as debt rises.¹ Distortionary taxes, however, limit the tax revenue that the government can generate. This allows us to derive the maximum level of debt that the government can repay, which we call the "effective debt limit". Solvency requires the debt level to remain below the effective debt limit. Adverse shocks, such as the 2008 global financial crisis or the 2020 global pandemic, could send debt above its limit. Beyond this effective debt limit, debt embarks on an explosive path, creditors refuse to lend, and the country faces a solvency crisis, forcing default to restore solvency. Default is due

¹ Bohn (1998) shows that a sustainable fiscal policy should respond positively to debt .

to the inability to repay as in Leeper and Walker (2011) and Bi (2012), and not due to a strategic decision as in Eaton and Gersovitz (1981) and Arellano (2008). To prevent a solvency crisis and default, debt needs to remain below this effective debt limit. As the debt level approaches the effective debt limit, expectations of default raise the interest rate, thereby pushing debt closer to its limit. A high-debt country could aggressively raise its tax rate with the hope to raise enough revenue to reduce debt. In this paper, we show that tax austerity not only affects the level of debt, but it also affects the position of the effective debt limit. When the tax rate increases close to the peak of the Laffer curve, tax austerity can raise the effective debt limit and avert a solvency crisis. Tax austerity, however, can backfire as it can lower the effective debt limit and trigger a crisis. This happens when the tax hikes push the tax rate to the slippery side of the Laffer curve. The first takeaway of our analysis is that tax austerity affects the position of the effective debt limit, thereby affecting the likelihood of a solvency crisis.² The second takeaway is that there is a nonlinear relationship between tax austerity and the effective debt limit, and equivalently between tax austerity and a solvency crisis.

Most of the austerity literature focuses on disentangling the effects of austerity on output. In traditional Keynesian models, austerity contracts aggregate demand and reduces output. In contrast, Giavazzi and Pagano (1990) suggest that austerity could be expansionary due to expectations. The impact of austerity, however, goes beyond output and the debt-to-GDP ratio. Tax austerity also affects the effective debt limit.

The literature offers two concepts for the maximum level of debt. Bi (2012) and Bi et al.

² Although models with default decisions under uncertainty can yield a state-dependent borrowing limit, to the best of our knowledge this is the first paper to show that the effective debt limit depends on tax austerity.

(2013) use a DSGE framework and derive their fiscal limit from the top of dynamic Laffer curves. Combining the peak of the Laffer curve with the government's intertemporal budget constraint, they obtain their fiscal limit on debt. Ghosh et al. (2013) identify their debt limit by estimating a cubic fiscal rule that describes the relationship between the primary surplus and debt. A negative coefficient on the cubic term captures the fiscal fatigue phenomenon and eventually yields an unstable region. Once debt enters the unstable region, debt embarks on an explosive path. The value of debt on the boundary of the unstable region represents their debt limit.

We draw on these models and propose an alternative procedure to derive the maximum level of debt consistent with solvency. One of the contributions of this paper is bridging the gap between the fiscal fatigue approach of Ghosh et al. (2013), which relies on unstable regions, and the Laffer curve approach of Bi (2012), which relies on distortionary taxes. The effective debt limit presented in this paper can be viewed as adding distortionary taxes in the Ghosh et al. (2013) model. By endogenizing output, the model naturally yields unstable regions.

Or alternatively, our paper can be viewed as extending Bi's (2012) model in two key ways. First, we relax the assumption that the peak of the Laffer curve can be attained instantaneously. Countries do not seem to immediately raise their tax rates to the maximum tax rate as politics interfere. Daniel and Shiamptanis (2022), among others, verify empirically that the government surplus exhibits substantial persistence, capturing the inertia in the legislative and implementation process. When we allow for persistence in the tax rate, we find that the effective debt limit depends on the value of the initial tax rate. We obtain a hump-shaped effective debt limit where the value of debt along the limit is higher for

medium tax rates than for low and high tax rates, implying that countries with either very low and very high initial tax rates can experience a solvency crisis at a lower level of debt. Second, we relax the assumption that the peak of the Laffer curve can be maintained forever. Countries might be unable or do not have the political power to maintain the maximum tax rate indefinitely. For example, Portugal, Greece and Ireland agreed to raise their tax rates as demanded by international creditors, but then stated that they could not maintain these high rates indefinitely. Additionally, countries being indefinitely at the peak of their Laffer curves is not in the data. Trabandt and Uhlig (2011) find evidence of countries being on either side of the Laffer curve. If a country cannot stay forever at the peak of the Laffer curve, we find that all the fiscal policy parameters, including the tax adjustment parameter, affect the position of the effective debt limit. By mapping all the fiscal policy parameters into the effective debt limit, this paper provides a tool to investigate the implications of tax austerity on solvency crisis.

Our paper is also related to Arellano and Bai (2016) who study the linkage between tax austerity and default. Our model is analogous to their fiscal default scenario, which occurs due to the inability of the government to raise tax revenue. In their paper, however, they find that in the presence of fiscal constraints, higher tax rates lower the likelihood of a solvency crisis. Similarly, Mendoza et al. (2014) who estimate dynamic Laffer curves and investigate the impact of tax rate increases for European countries also find that higher labour tax rates improve solvency. In contrast, we find a nonlinear relationship between tax austerity and the probability of a solvency crisis. Bianchi et al. (2019) consider the effects of spending cuts and show that aggressive spending cuts can backfire as they raise the incentives to default, but they abstract from debt limits. Our paper focuses on tax austerity and its effect on the

effective debt limit.

The final contribution of the paper is quantitative. We apply our model to Italy, a country which is under ongoing pressure to rein its elevated debt level. We estimate the Italian effective debt limit and quantify the probability of solvency crisis. Next we use our model to ask whether tax austerity could alter solvency risk in Italy. We find that if Italy adopts tax austerity to lower its debt, it will also lower its effective debt limit, thereby raising the danger of future insolvency.

This paper is organized as follows. Section 2 presents the model, derives the effective debt limit and shows that it depends on tax austerity. Section 3 applies the model to Italy and Section 4 provides conclusions.

2 Model

We set up a small open economy model. The country faces an effective debt limit, which arises endogenously as the debt dynamics interact with the tax rate dynamics because of a Laffer curve effect. The effective debt limit is the maximum level of debt consistent with solvency, conditional on the fiscal rules in place. If adverse shocks push the debt level above its limit, the country faces a solvency crisis and defaults. Default is due to insolvency. We assume that a solvent government always repays.

2.1 Government

The domestic government issues bonds (b_t) which can be held by the domestic agent (b_t^d) or the foreign agent (b_t^f), such that $b_t = b_t^d + b_t^f$.³ The government's real flow budget constraint

³ We depart from Mendoza et al. (2014) who assume that the domestic government bonds are held entirely by the domestic agent ($b_t = b_t^d$), and from Bi et al. (2016) and Arellano and Bai (2016) who assume that

is given by

$$b_t = (1 + i_{t-1}) \delta_t b_{t-1} - \tau_t y_t + g_t + z_t. \quad (1)$$

where i_{t-1} is the interest rate that domestic bonds pay, g_t denotes government purchases, z_t is the transfer payments to the household, τ_t represents the distortionary labour income tax, y_t is the output, which depends on the household's optimization behaviour and labour decisions, and δ_t represents the repayment rate on government bonds, such that $\delta_t = 1$ implies no default, and $\delta_t < 1$ implies partial default on both the domestic and foreign bond holders. The equilibrium value of δ_t depends on the distance between the effective debt limit and the debt level, and it is endogenously determined below.

We assume that the foreign agent is willing to buy the domestic government bonds (b_t^f) as long as the domestic interest rate (i_{t-1}) satisfies interest rate parity. Interest rate parity is derived from the foreign agent's Euler equations when the covariance between the domestic interest rate and the foreign agent's consumption is zero,⁴ and it can be expressed as

$$1 + i = (1 + i_{t-1}) E_{t-1} \delta_t \quad (2)$$

where i is the foreign default-free interest rate, which is assumed to be constant. Equation (2) implies that the domestic interest rate (i_{t-1}) rises above the foreign interest rate (i) when the agents expect the government to default ($E_{t-1} \delta_t < 1$).⁵ If there are no expectations for default ($E_{t-1} \delta_t = 1$), then the domestic interest rate is equal to the foreign interest rate ($i_{t-1} = i$).

domestic government bonds are held entirely by the foreign agent ($b_t = b_t^f$).

⁴ Equation (2) also holds when the foreign agent is risk neutral.

⁵ Expectations of default ($E_{t-1} \delta_t$) arise as the debt level approaches the effective debt limit. The equilibrium value depends on the distance between the effective debt limit and the debt level, and it is endogenously determined below.

Define the capital loss on debt due to default, denoted by a_t , as

$$a_t = (1 - \delta_t)(1 + i_{t-1})b_{t-1} \quad (3)$$

where default ($\delta_t < 1$) increases the capital loss on debt ($a_t > 0$).⁶ Using equation (3), unexpected capital loss due to default can be expressed as

$$a_t - E_{t-1}a_t = (E_{t-1}\delta_t - \delta_t)(1 + i_{t-1})b_{t-1}. \quad (4)$$

Substituting equations (2) and (4) into equation (1) yields the evolution of government's debt as

$$b_t = (1 + i)b_{t-1} - \tau_t y_t + g_t + z_t - (a_t - E_{t-1}a_t) \quad (5)$$

where expectations of capital loss ($E_{t-1}a_t > 0$) raise debt, and capital loss due to default ($a_t > 0$) reduces debt. Default provides revenue only if it is larger than its expected value. When default is fully anticipated, it raises both a_t and $E_{t-1}a_t$ equally, thereby having no effect on debt ($a_t - E_{t-1}a_t = 0$). Equation (5) allows to linearly separate the terms that affect the domestic interest rate.

2.2 Fiscal policy rules

We specify simple exogenous fiscal rules. The fiscal rules are analogous to the Taylor rule commonly used for monetary policy. Perhaps the most successful characterizations of fiscal policy are due to Leeper (1991) and Bohn (1998). They find that simple fiscal rules that respond to debt describe the behaviour of fiscal authorities quite well. Bohn (1998), Mendoza and Ostry (2008), and a very large empirical literature specify fiscal policy as a rule in which the primary surplus adjusts to debt and find a positive response to debt.⁷ Similarly, Leeper

⁶ When there is no default ($\delta_t = 1$), there is no capital loss on debt ($a_t = 0$).

(1991, 2010), Bi (2012), Bi et al. (2013), Leeper et al. (2010) utilize simple rules and find that the fiscal authorities increase the tax rate as debt rises. This behaviour appears to be consistent with the casual empirical evidence offered in Figure 1. The scatter plot illustrates the positive relationship between the Italian tax rate and debt, suggesting that high tax rates are associated with high debt levels. Our tax rule generalizes those used by Bi (2012) and Bi et al. (2013) by allowing the tax rate (τ_t) to respond to its own lag (τ_{t-1}), in addition to lagged debt (b_{t-1}). The tax rule is given by

$$\tau_t - \tau = \rho^\tau (\tau_{t-1} - \tau) + \gamma (b_{t-1} - b) \quad (6)$$

where τ and b are the steady-state values of the tax rate and debt, respectively, ρ^τ measures the persistence in the tax rate, which partly captures the inertia in the legislative and implementation process, and partly reflects the desire to smooth the effects of debt deviations from its steady-state over time. The coefficient γ is the tax adjustment parameter, which captures the responsiveness of the tax rate to increases in debt and is our main measure for austerity. We refer to an increase in γ as "tax austerity". A large γ suggests that the government is aggressively raising the tax rate as debt rises, whereas a small γ suggests a weak response by the government to increases in debt. The magnitude of γ plays a key role in policy decisions as it provides evidence that the government is taking actions as debt rises (Bohn 1998).⁸

We specify government purchases (g_t) and transfers (z_t) as AR(1) processes for two reasons. First, the processes capture the Italian behaviour over our sample period. The

Bohn (1998, 2008) find that the primary surplus in the US responds positively to debt. Similarly, Mendoza and Ostry (2008) find a positive response to debt in a panel of developed and developing countries. D'Erasmus et al. (2016) provide an overview of the empirical fiscal sustainability literature.

⁸ Bohn (1998) shows that a positive γ is sufficient for the intertemporal government budget constraint to hold. Leeper (1991) shows that γ should be positive and sufficiently large to keep debt bounded.

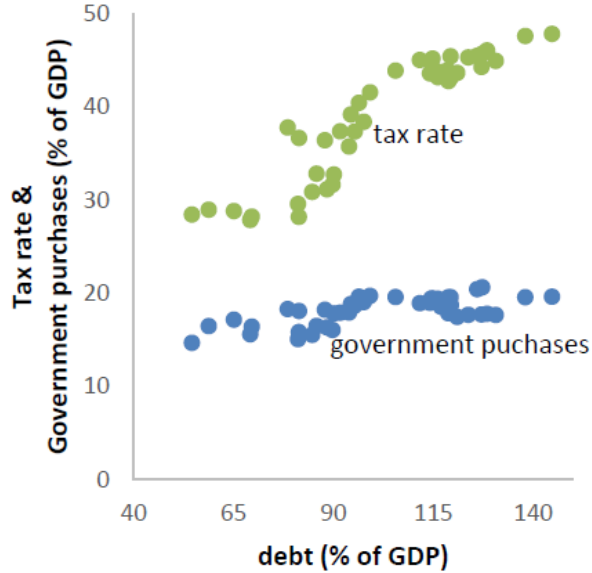


Figure 1: *Scatter plot for Italy between 1970 and 2015.*

scatter plot in Figure 1 shows that government purchases are around 18% of GDP, regardless of the level of debt. We find that Italian fiscal authorities do not cut government purchases or transfers in response to increases in debt.⁹ Second, we follow Bi (2012) who also specifies government purchases and transfers as exogenous processes. We, therefore, specify them as

$$g_t - g = \rho^g (g_{t-1} - g) + \varepsilon_t^g, \quad \varepsilon_t^g \sim N(0, \sigma^g) \quad (7)$$

$$z_t - z = \rho^z (z_{t-1} - z) + \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0, \sigma^z) \quad (8)$$

where g and z represent the steady-state values of government purchases and transfers, respectively. The parameters ρ^g and ρ^z capture persistence, and ε_t^g and ε_t^z represent random and unanticipated shocks. Although we do not find evidence that the Italian authorities lower government purchases or transfers as debt rises, we investigate the implications of lower steady-state values of government purchases (g) and transfers (z). Lower values for

⁹ When we add b_{t-1} in the government purchases and transfers equations, the estimated coefficients are not negative.

g and z capture the declines in the government size and represent some of the measures recently adopted by some European countries.

2.3 Household

The domestic small open economy is populated by a representative household who maximizes the following utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log c_t + \phi \log l_t],$$

subject to the following budget constraint

$$b_t^d + b_t^{*d} = (1 + i_{t-1}) \delta_t b_{t-1}^d + (1 + i) b_{t-1}^{*d} + (1 - \tau_t) y_t + z_t - c_t \quad (9)$$

where c_t is consumption, l_t is leisure, ϕ is a preference leisure parameter, $0 < \beta < 1$ is the discount factor, b_t^d is the domestic government bond, b_t^{*d} is the international bond, i_{t-1} is the domestic interest rate, δ_t is the repayment rate of the domestic bonds, i is the foreign interest rate, τ_t is the tax rate, z_t is the transfer payments and y_t is output. E_t denotes the expectation conditional on the information at time t .

Output (y_t) is determined by the productivity level (A_t) and the labour supply ($1 - l_t$),

$$y_t = A_t (1 - l_t)$$

where the productivity level follows an AR(1) process with A representing the steady-state level and ε_t^A the productivity shocks

$$A_t - A = \rho^A (A_{t-1} - A) + \varepsilon_t^A, \quad \varepsilon_t^A \sim N(0, \sigma^A).$$

Combining the government budget constraint (equation 1) with the household budget constraint (equation 9), the aggregate resource constraint is given by

$$A_t (1 - l_t) - c_t - g_t = (1 + i_{t-1}) \delta_t b_{t-1}^f - b_t^f + b_t^{*d} - (1 + i) b_{t-1}^{*d} \quad (10)$$

where the right hand side of equation (10) represents the trade balance, which is assumed to be proportional to the primary balance¹⁰

$$(1 + i_{t-1}) \delta_t b_{t-1}^f - b_t^f + b_t^{*d} - (1 + i) b_{t-1}^{*d} = \lambda (\tau_t y_t - g_t - z_t).$$

The household's maximization problem yields the typical first-order conditions and the output can be written as

$$y_t = \frac{A_t (1 - \tau_t) + \phi (1 - \lambda) g_t - \phi \lambda z_t}{1 + \phi - (1 + \phi \lambda) \tau_t}. \quad (11)$$

An increase in τ_t lowers output ($\frac{\partial y_t}{\partial \tau_t} < 0$), and this allows for the Laffer curve to endogenously arise in the model. Using equation (11), the tax rate at the peak of the Laffer curve, denoted by τ_t^L , can be written as

$$\tau_t^L = \frac{2(1 + \phi) A_t - \sqrt{4(1 + \phi)^2 A_t^2 - 4(A_t + \phi(1 - \lambda) g_t - \phi \lambda z_t)(1 + \phi)(1 + \phi \lambda) A_t}}{2(1 + \phi \lambda) A_t}. \quad (12)$$

2.4 Dynamics

It is useful to represent the dynamic behaviour of the expected future values of the tax rate and debt using a phase diagram, which reveals the direction of movement of the tax rate and debt for different initial values. We construct the phase diagram for the system by subtracting the lagged value of the tax rate from both sides of equation (6) and the lagged

¹⁰This is a simplifying assumption that does not require modeling the foreign country. Our key result is robust to alternative assumptions. If we model the foreign country, then our effective debt limit will also depend on the foreign country's trade balance. But, the key findings of our paper remain unchanged.

The default literature usually considers two special cases: $\lambda = 0$ and $\lambda = 1$. Closed economy default models (as in Bi 2012) assume that the domestic government bonds are held entirely by the domestic agent ($b_t = b_t^d$), which in turn implies that $\lambda = 0$. Small open economy default models (as in Arellano and Bai 2016, Bi et al. 2016) assume that the domestic government bonds are held entirely by the foreign agent ($b_t = b_t^f$) and that the domestic agent does not hold any international bonds ($b_t^{*d} = 0$). This traditional assumption in small open economy models implies that the trade balance is equal to the primary balance (i.e. $\lambda = 1$). In this paper, we depart from this traditional assumption and investigate the implications when the trade balance is not identical to the primary balance (i.e. λ takes any value other than unity). In the Appendix, we show the effect on our effective debt limit when $\lambda = 0$ and $\lambda = 1$.

value of debt from both sides of equation (5), taking the equations j periods forward, and taking the time t expectation to yield

$$E_t \tau_{t+j} - E_t \tau_{t+j-1} = (\rho^\tau - 1)(E_t \tau_{t+j-1} - \tau) + \gamma(E_t b_{t+j-1} - b) \quad (13)$$

$$\begin{aligned} E_t b_{t+j} - E_t b_{t+j-1} &= i E_t b_{t+j-1} - E_t \left(\frac{A_{t+j} \tau_{t+j} (1 - \tau_{t+j}) + \phi (1 - \lambda) g_{t+j} \tau_{t+j} - \phi \lambda z_{t+j} \tau_{t+j}}{1 + \phi - (1 + \phi \lambda) \tau_{t+j}} \right) \\ &\quad + E_t g_{t+j} + E_t z_{t+j} + E_t (a_{t+j} - E_{t+j-1} a_{t+j}). \end{aligned} \quad (14)$$

where $E_t (a_{t+j} - E_{t+j-1} a_{t+j}) = 0$.¹¹ Equations (13) and (14) do not rule out default (a_{t+j}) or expectations of default ($E_{t+j-1} a_{t+j}$). They can still occur, but they do not provide expected and systematic revenue or expenditure.

Setting equations (13) and (14) equal to zero ($\Delta E_t \tau_{t+j} = 0$, $\Delta E_t b_{t+j} = 0$), the two equations for the phase diagram are given by

$$E_t b_{t+j-1} = \frac{\gamma b + (\rho^\tau - 1) \tau + (1 - \rho^\tau) E_t \tau_{t+j}}{\gamma} \quad (15)$$

$$\begin{aligned} E_t b_{t+j-1} &= E_t \left(\frac{A_{t+j} \tau_{t+j} (1 - \tau_{t+j}) + \phi (1 - \lambda) g_{t+j} \tau_{t+j} - \phi \lambda z_{t+j} \tau_{t+j}}{i (1 + \phi - (1 + \phi \lambda) \tau_{t+j})} \right) \\ &\quad - E_t \left(\frac{g_{t+j} + z_{t+j}}{i} \right). \end{aligned} \quad (16)$$

The phase diagram is presented in Figure 2. The debt is on the vertical axis and the tax rate is on the horizontal axis. The $\Delta E_t \tau_{t+j} = 0$ curve, equation (15), is linear and it has a positive slope, $\frac{1 - \rho^\tau}{\gamma} > 0$. The $\Delta E_t b_{t+j} = 0$ curve, equation (16), is nonlinear and its shape mimics the shape of the Laffer curve. The peak of the $\Delta E_t b_{t+j} = 0$ curve occurs at the tax rate τ_t^L , equation (12). The two curves intersect at points G and H. Point G represents the long-run equilibrium in which the tax rate and debt are equal to their steady-state values ($\tau^G = \tau$, $b^G = b$). Using equation (16), the steady-state value of debt is given by

$$b = \frac{\tau y - g - z}{i} \quad (17)$$

¹¹ $E_t (a_{t+j} - E_{t+j-1} a_{t+j}) = E_t a_{t+j} - E_t (E_{t+j-1} a_{t+j}) = E_t a_{t+j} - E_t a_{t+j} = 0$

where $\tau y - g - z$ is the steady-state value of the primary surplus. At point G, the primary surplus is equal to interest on debt (ib). Equations (15) and (16) divide the system into different regions, with the arrows of motion revealing the direction of movement of debt and the tax rate in each region. Adverse shocks push the economy away from its long-run equilibrium. If a shock pushes the economy to point A, then the economy is expected to travel along the adjustment path AG. It illustrates how the system is expected to travel in the absence of additional shocks. The adjustment path AG crosses three regions. In the initial region, both the tax rate and debt are rising as illustrated by the arrows of motion. In the second region, the tax rate continues to rise while the debt level is gradually falling. In the third region, both the tax rate and debt are declining, and the economy eventually reaches point G.

Dividing equation (14) by equation (13) yields the time-varying slope of any adjustment path as

$$\frac{\Delta E_t b_{t+j}}{\Delta E_t \tau_{t+j}} = \frac{i E_t b_{t+j-1} + E_t g_{t+j} + E_t z_{t+j}}{(\rho^\tau - 1) (E_t \tau_{t+j-1} - \tau) + \gamma (E_t b_{t+j-1} - b)} \quad (18)$$

$$- \frac{E_t \left(\frac{A_{t+j} \tau_{t+j} (1 - \tau_{t+j}) + \phi (1 - \lambda) g_{t+j} \tau_{t+j} - \phi \lambda z_{t+j} \tau_{t+j}}{i (1 + \phi - (1 + \phi \lambda) \tau_{t+j})} \right)}{(\rho^\tau - 1) (E_t \tau_{t+j-1} - \tau) + \gamma (E_t b_{t+j-1} - b)},$$

which can be positive or negative depending on the values of the tax rate and debt.

Point H is the second intersection point between equations (15) and (16), and the values of the tax rate and debt are given respectively by

$$\tau^H = 1 - \tau + \frac{((1 - \rho^\tau) \tau - \gamma b) i (1 + \phi \lambda) - g \gamma (1 + \phi) - z \gamma + (1 - \rho^\tau) i \phi (1 - \lambda)}{(1 - \rho^\tau) i (1 + \phi \lambda) - \gamma A} \quad (19)$$

$$b^H = b + \frac{(\rho^\tau - 1) \tau + (1 - \rho^\tau) \tau^H}{\gamma}, \quad (20)$$

which depend among other parameters on tax austerity (γ).

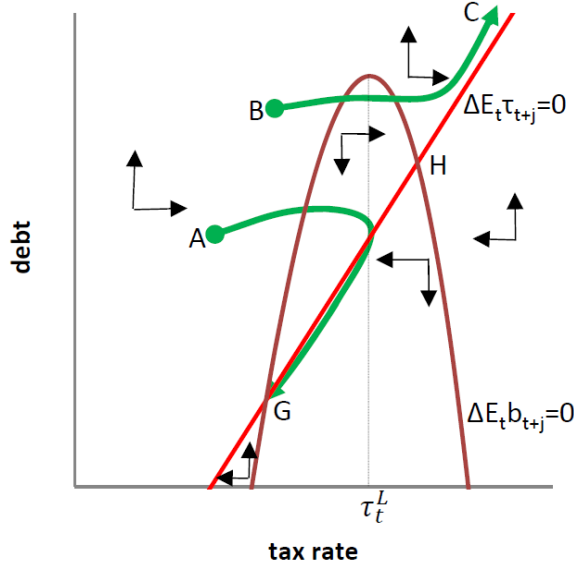


Figure 2: *Phase diagram*

If shocks are so large that push the economy north of point H, then the debt embarks on an explosive path and thus fails to attain its long-run equilibrium, as illustrated by path BC.¹² Explosive paths do not represent equilibrium paths. This is a locally stable model, implying that the economy is expected to reach its long-run equilibrium (point G) for only some values of the tax rate and debt.

2.5 Effective debt limit

We exploit the unstable regions in our model and derive the maximum values of debt consistent with solvency, conditional on the fiscal rules in place. There is a saddlepath relationship between the tax rate and debt towards point H. The saddlepath, labelled DEH in Figure 3, is the boundary of the stable region that separates paths that converge to the long-run equilibrium and those that do not. Beginning at any position below DEH, the economy is expected to reach its long-run equilibrium (point G). The adjustment path AG is an exam-

¹²Although the BC path enters the region below the $\Delta E_t b_{t+j} = 0$ curve in which debt slightly falls, it exits this region and debt becomes explosive.

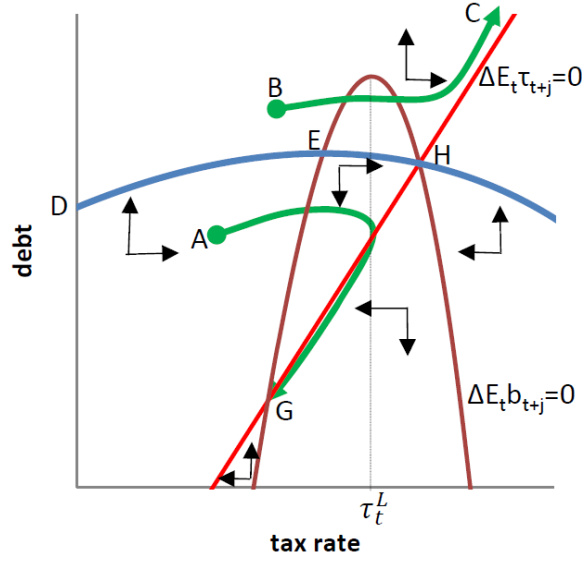


Figure 3: *Effective debt limit*

ple of path that returns to the long-run equilibrium in the absence of additional shocks. If shocks push the debt above DEH, the primary surplus is not sufficient to pay the interest payments and debt becomes explosive. This is a position of insolvency as creditors would refuse to lend to government whose debt is explosive, implying that any paths above DEH, such as BC, are infeasible. Therefore, in equilibrium the system should not exceed DEH and this makes the saddlepath our effective debt limit.

We approximate the value for debt along the saddlepath DEH, which we label as \hat{b}_t , by taking a piecewise linear approximation of this path about \hat{b}_{t-1} and τ_{t-1} , and using equations (6) and (18) to yield

$$\hat{b}_t = \hat{b}_{t-1} + \hat{\zeta}_{t-1} (\tau_t - \tau_{t-1}), \quad (21)$$

where $(\tau_t - \tau_{t-1})$ is the change in the tax rate, equation (6), and $\hat{\zeta}_{t-1}$ is the slope of the

saddlepath DEH,

$$\hat{\zeta}_{t-1} = \frac{i\hat{b}_{t-1} + (1 - \rho^g)g + \rho^g g_{t-1} + (1 - \rho^z)z + \rho^z z_{t-1}}{(\rho^\tau - 1)(\tau_{t-1} - \tau) + \gamma(\hat{b}_{t-1} - b)} - \left[(1 - \rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma \hat{b}_{t-1} \right] \bullet$$

$$\frac{\left[\frac{((1 - \rho^A)A + \rho^A A_{t-1})(1 - (1 - \rho^\tau)\tau + \gamma b - \rho^\tau \tau_{t-1} - \gamma \hat{b}_{t-1}) + \phi(1 - \lambda)((1 - \rho^g)g + \rho^g g_{t-1}) - \phi\lambda((1 - \rho^z)z + \rho^z z_{t-1})}{1 + \phi - (1 + \phi\lambda)((1 - \rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma \hat{b}_{t-1})} \right]}{(\rho^\tau - 1)(\tau_{t-1} - \tau) + \gamma(\hat{b}_{t-1} - b)}$$

The slope is positive ($\hat{\zeta}_{t-1} > 0$) when the tax rate and debt are rising along DE, $\hat{\zeta}_{t-1} = 0$ once the effective debt limit reaches its peak at point E, and the slope is negative ($\hat{\zeta}_{t-1} < 0$) beyond point E. Using the saddlepath combination of debt and the tax rate at point H,¹³ we trace out the values of debt along the effective debt limit for each value of the tax rate.

Our effective debt limit is nonlinear with values depending on the level of the tax rate. For low values of the tax rate, the effective debt limit is upward-sloping until it peaks at point E, which is the first intersection point between the effective debt limit, equation (21), and the $\Delta E_t b_{t+j} = 0$ curve, equation (16).¹⁴ For larger values of the tax rate beyond point E, the critical boundary is downward-sloping. Our hump-shaped effective debt limit implies that a country could experience a solvency crisis at different levels of debt.¹⁵ When the tax rate is either too low (near point D) or too high (beyond point H), the values of debt along the effective debt limit are lower, implying that a country could experience a solvency crisis at lower levels of debt.¹⁶

¹³ $\tau_{t-1} = \tau^H$ and $\hat{b}_{t-1} = b^H$

¹⁴Our results reveal that the peak of our effective debt limit (point E) does not necessarily occur at τ_t^L , suggesting that a country does not have to raise its tax rate to τ_t^L to reach the maximum value of debt consistent with solvency.

¹⁵Persistence in the tax rate (ρ^τ) is critical for the hump-shaped specification. As ρ^τ approaches zero, the effective fiscal limit becomes horizontal ($\lim_{\rho^\tau \rightarrow 0} \hat{\zeta}_{t-1} \rightarrow 0$) and goes through point H. For more details see Appendix A

¹⁶Our effective debt limit represents the maximum value of debt that can be supported in the absence of expected and systematic revenue or expenditure from default. Default and expectations of default do not affect our effective fiscal limit as they are equal in expectations, $E_t(a_{t+j} - E_{t+j-1}a_{t+j}) = 0$.

2.6 Solvency crisis resolved with default

Definition: *Equilibrium in the market for domestic government bonds:* Given the foreign interest rate (i), fiscal policy parameters (γ, τ, g, z), initial values ($b_{t-1}, \tau_{t-1}, g_{t-1}, z_{t-1}, A_{t-1}$), stochastic processes for ε_t^A , ε_t^g and ε_t^z , and the dynamic equation for the tax rate (equation 6), an equilibrium is values for $\{E_{t-1}\delta_t, E_{t-1}a_t, b_t, \tau_t, g_t, z_t, A_t, y_t, i_t, \delta_t, a_t\}$, such that expectations are rational, international creditors expect to receive i on domestic government bonds (equation 2), the government's flow budget constraint (equation 5) is satisfied, and the debt does not exceed its effective debt limit (equation 21)■

Solvency requires that the debt level remains below its effective debt limit (\hat{b}_t). We assume that a solvent government fully repays and there is no default ($\delta_t = 1$).¹⁷ If, however, adverse shocks push the debt beyond its effective debt limit, agents refuse to lend and there is a solvency crisis. To restore lending, the government reduces debt via default to the effective debt limit, which is the highest value of debt consistent with solvency. This implies that default is never 100% ($\delta_t \neq 0$). This is an alternative type of default, not the strategic type of default commonly analyzed in the literature of sovereign default (Eaton and Gersovitz 1981; Arellano 2008), but due to inability to repay (Bi 2012; Bi et al. 2013, 2018; Daniel and Shiamptanis 2012). This is a model with involuntary default where partial default occurs when the government's solvency constraint cannot be satisfied.

We write the fiscal space, Ω_t , between the effective debt limit, equation (21), and the

¹⁷Recall that δ_t represents the repayment rate, such that $\delta_t = 1$ implies no default, $\delta_t = 0$ implies 100% default, and $\delta_t < 1$ implies partial default.

value of debt, equation (5), as

$$\begin{aligned}\Omega_t &= \hat{b}_t - b_t \\ &= x_{t-1} + u_t + a_t - E_{t-1}a_t\end{aligned}\tag{22}$$

where x_{t-1} is the state variable determining the distance between the effective debt limit and debt, and is given by

$$\begin{aligned}x_{t-1} &= \hat{b}_{t-1} + \hat{\zeta}_{t-1} ((\rho^\tau - 1) (\tau_{t-1} - \tau) + \gamma (b_{t-1} - b)) - (1 + i) b_{t-1} + \\ &\quad \frac{\tau_t [((1 - \rho^A) A + \rho^A A_{t-1}) (1 - \tau_t) + \phi (1 - \lambda) ((1 - \rho^g) g + \rho^g g_{t-1}) - \phi \lambda ((1 - \rho^z) z + \rho^z z_{t-1})]}{1 + \phi - (1 + \phi \lambda) \tau_t} \\ &\quad - ((1 - \rho^g) g + \rho^g g_{t-1}) - ((1 - \rho^z) z + \rho^z z_{t-1}),\end{aligned}\tag{23}$$

u_t is the total impact of the productivity and fiscal shocks on the fiscal space

$$u_t = \frac{\tau_t [\varepsilon_t^A (1 - \tau_t) + \phi (1 - \lambda) \varepsilon_t^g - \phi \lambda \varepsilon_t^z]}{1 + \phi - (1 + \phi \lambda) \tau_t} - \varepsilon_t^g - \varepsilon_t^z$$

a_t represents the magnitude of capital loss due to default, and $E_{t-1}a_t$ is the expectations of capital loss due to default, which are derived in the online Appendix.

If the state variable x_{t-1} is sufficiently large such that no shocks (u_t) will push the economy over its effective debt limit, then the agents are not expecting default ($E_{t-1}\delta_t = 1$) and the expectations of capital loss are zero ($E_{t-1}a_t = 0$). Alternatively, if x_{t-1} is relatively small, agents expect default ($E_{t-1}\delta_t < 1$), which then increases the expectations of capital loss ($E_{t-1}a_t > 0$). The magnitudes of $E_{t-1}\delta_t$ and $E_{t-1}a_t$ depend on x_{t-1} , which in turn depend among other parameters on tax austerity (γ). The combination of the endogenous expectations of capital loss due to default ($E_{t-1}a_t > 0$) and negative stochastic shocks ($u_t < 0$) can push debt (b_t) above its effective debt limit (\hat{b}_t), such that the fiscal space

becomes negative ($\Omega_t < 0$) in equation (22). To restore the equilibrium, the government partially defaults ($\delta_t < 1$).¹⁸ The magnitude of capital loss ($a_t > 0$) returns debt back to the effective debt limit, implying that in the next period $x_t = 0$.

After default, there are subsequent future defaults and output losses, but for different reasons than the ones considered in the strategic default models. In our model, after default there are no international trade sanctions and no reputational costs. Agents do not coordinate to exclude the country from the international financial markets. Daniel and Shiamptanis (2012) show that in the next period when the difference between the effective debt limit and debt is zero ($x_t = 0$), the expectations of capital loss ($E_t a_{t+1} > 0$) are elevated, such that additional capital loss ($a_{t+1} > 0$) is necessary to set $x_{t+1} = 0$. During this turbulent period, the economy is moving along the effective debt limit DEH towards point H with additional defaults and higher tax rates. This pattern persists until the economy nears point H, where the dynamics eventually imply that future debt falls below its effective debt limit. In our model, the protracted contraction in output in the run-up and aftermath of default is endogenous and is steaming from the higher distortionary taxes ($\frac{\partial y_t}{\partial \tau_t} < 0$), rather than an exogenous cost function.

2.7 Implications of tax austerity

The effective debt limit is derived under the assumption that the country follows the fiscal rules given by equations (6) - (8) and does not adjust its policy parameters. Of course, a country can adjust its parameters.¹⁹ It is, therefore, important to investigate the effects of

¹⁸The detailed proofs and derivations are provided in the online Appendix.

¹⁹While most of the fiscal literature uses fixed fiscal rules, there are a few exceptions by Davig and Leeper (2011) and Daniel and Shiamptanis (2022) who estimate fiscal rules where the parameters on these rules can be different in different regimes.

different parameters on the effective debt limit and the likelihood of a solvency crisis. We begin by considering the effects of tax austerity. We represent tax austerity with an increase in the value of γ , and show that its magnitude affects the position of our effective debt limit. There are two cases. An increase in γ could either raise or lower the effective debt limit, depending on the position of point H.

Consider the case where the initial γ is large enough such that $\tau^H > \tau_t^L$. In Figure 4, a further increase in γ reduces the slope of the $\Delta E_t \tau_{t+j} = 0$ curve and rotates it clockwise around point G, moving point H to the right ($\frac{\partial \tau^H}{\partial \gamma} > 0$) and downwards ($\frac{\partial b^H}{\partial \gamma} < 0$), and thus increasing the area that the dynamics become explosive. Therefore, an increase in γ lowers the effective debt limit DEH, as illustrated in Figure 4. The implementation of tax austerity pushes a high-debt country at point A into insolvency. At the initial value of γ , point A is solvent as it is below the original effective debt limit. But if a high-debt country increases its γ , then it makes point A insolvent as it is above the new effective debt limit. High-debt countries with large initial tax responsiveness to debt (γ) that intensify their efforts to raise the tax rate are more prone to find themselves on the wrong side of the Laffer curve, where higher tax rates lower the tax revenue and the effective debt limit, triggering a solvency crisis. In this case, tax austerity backfires.

Next we consider the case where the initial γ is small enough such that $\tau^H < \tau_t^L$. In Figure 5, an increase in γ rotates the $\Delta E_t \tau_{t+j} = 0$ curve clockwise around point G and moves point H in the northeast direction ($\frac{\partial \tau^H}{\partial \gamma} > 0, \frac{\partial b^H}{\partial \gamma} > 0$), reducing the area in which the dynamics become explosive in favour of an increase in the area in which debt converges to its long-run equilibrium. In this case, the increase in γ raises the effective debt limit and reduces the likelihood a solvency crisis. The implementation of tax austerity helps a high-debt country

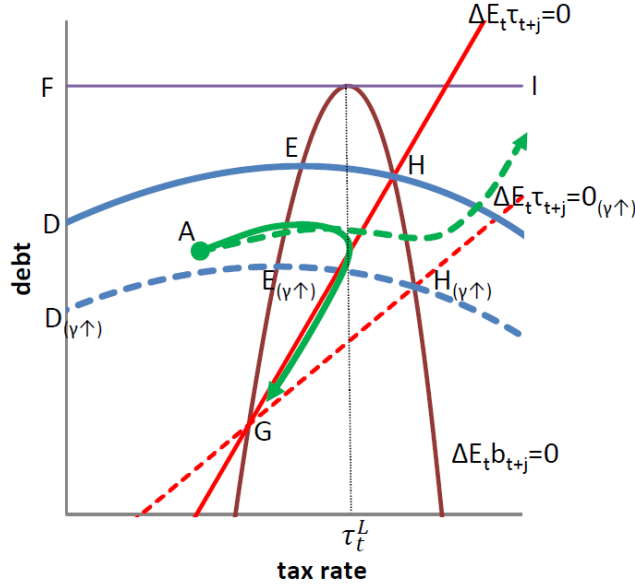


Figure 4: *Lower effective fiscal limit due to tax austerity*

at point B move into solvency. At the initial value of γ , point B is insolvent as it is above the original effective debt limit. An increase in γ makes point B solvent as it is below the new effective debt limit and the new adjustment path BG illustrates how the economy is expected to move in the absence of additional shocks. High-debt countries with small tax adjustment parameters that become aggressive in raising their tax rates could raise their effective debt limit, thereby averting or exiting a solvency crisis. In this case, tax austerity is successful.

To summarize, our results imply that the efficacy of tax austerity depends on the value of the tax adjustment parameter (γ) and that there is a nonlinear relationship between tax austerity and solvency crisis. When the value of γ is small, tax austerity could be successful in averting a solvency crisis in a high-debt country. But when the value of γ is already large, tax austerity could cause a solvency crisis in a high-debt country.²⁰ Our results could

²⁰In Figures 4 and 5, Bi's (2012) fiscal limit is represented by the horizontal line FI, which is tangent to the peak of the $\Delta E_t b_{t+j} = 0$ curve, implying that Bi's fiscal limit is the same, regardless of τ_t and γ .

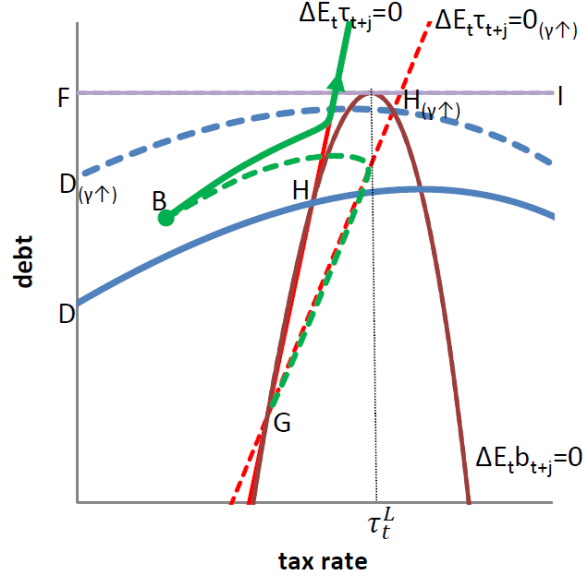


Figure 5: *Higher effective fiscal limit due to tax austerity*

shed some light on the ongoing debate in Europe about tax austerity in high-debt countries. Pressure on high-debt countries by EMU and IMF to adopt tax austerity could indicate the perception that the high-debt countries have a small γ such that further increases in γ can avert a solvency crisis, whereas the resistance by the high-debt countries to adopt tax austerity could indicate their judgement that they already have a large enough γ such that further increases in γ will result to insolvency.

2.7.1 Other types of austerity

A faster tax adjustment to debt (γ) is one policy option. The values of τ , g and z represent the size of the fiscal authority and can also be policy choices. The values that countries choose represent their risk appetite. We consider the effects of higher steady-state tax rate (τ) and lower steady-state values of government purchases (g) and transfers (z). We show that the position of the effective debt limit and the likelihood of a solvency crisis is also affected by the values of τ , g and z .

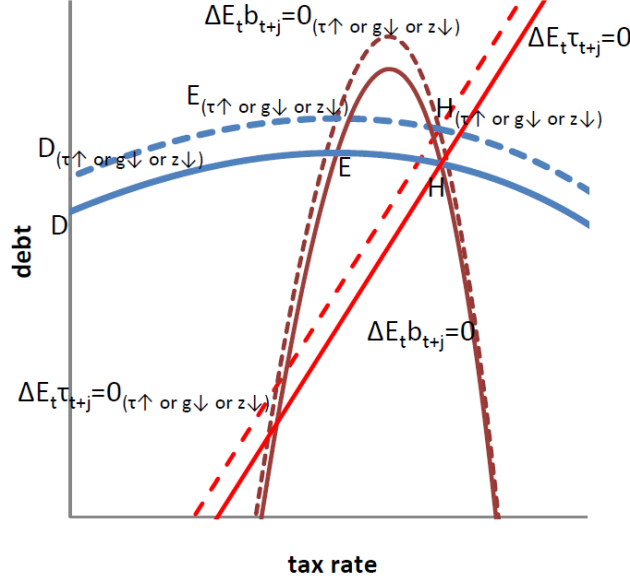


Figure 6: Higher effective fiscal limit due to higher τ , or lower g or lower z .

Using the long-run equilibrium, equation (17),

$$\tau y - g - z = ib$$

the steady-state value of primary surplus ($\tau y - g - z$) must be equal to the interest payments (ib). This implies that changes in the steady-state fiscal variables (τ, g or z) will require a change in the country's steady-state debt (b) to satisfy equation (17).²¹ The values of τ, g, z and b should be consistent with each other given the foreign interest rate (i). A larger τ or smaller g or smaller z raise both the steady-state values of the primary surplus and debt, and in turn shift both the $\Delta E_t \tau_{t+j} = 0$ and $\Delta E_t b_{t+j} = 0$ curves upwards, raising the effective debt limit as shown in Figure 6. Our results imply that higher τ , or lower g and z can be successful in averting a solvency crisis.

²¹The foreign interest rate (i) is outside the domestic country's control and does not adjust to the domestic policy. This follows from the small open economy assumption.

3 Model Applied: The case of Italy

In this section, we apply the model to Italy, a country with the second highest debt level in Europe. First, we estimate the effective debt limit implied by our model, and we then compare our results with Bi’s (2012) fiscal limit and Ghosh et al.’s (2013) debt limit.²²

Second, we quantify the probability of a solvency crisis when tax austerity is implemented.

Table 1: Calibration to the Italian economy

Parameters	Value
Discount factor (β)	0.9615
Labour ($1 - l$)	0.25
Leisure preference parameter (ϕ)	2.21
Technology (A)	1
Tax rate (τ)	0.40
Government spending/GDP (g/y)	0.18
Transfers/GDP (z/y)	0.20
Debt/GDP (b/y)	0.50
trade balance/primary balance (λ)	0.216
Persistence of taxes (ρ^τ)	0.63
Tax adjustment (γ)	0.34
Persistence of government spending (ρ^g)	0.69
Standard deviation of technology (σ^g)	0.0011
Persistence of technology (ρ^A)	0.44
Standard deviation of technology (σ^A)	0.0150
Persistence of transfers (ρ^z)	0.31
Standard deviation of transfers (σ^z)	0.0012

3.1 At what point could Italy become insolvent?

In this section, we quantify the Italian effective debt limit. Table 1 summarizes our parameter values. The model is calibrated at annual frequency. The steady-state fiscal variables are calibrated to match the average Italian data between 1970 and 2015: the tax rate (τ) is 0.40, the government spending-to-GDP ratio (g/y) is 0.18, and the transfer payments-to-

²²The online Appendix provides a detailed comparison between our effective debt limit and the limits proposed by Bi (2012) and Ghosh et al. (2013).

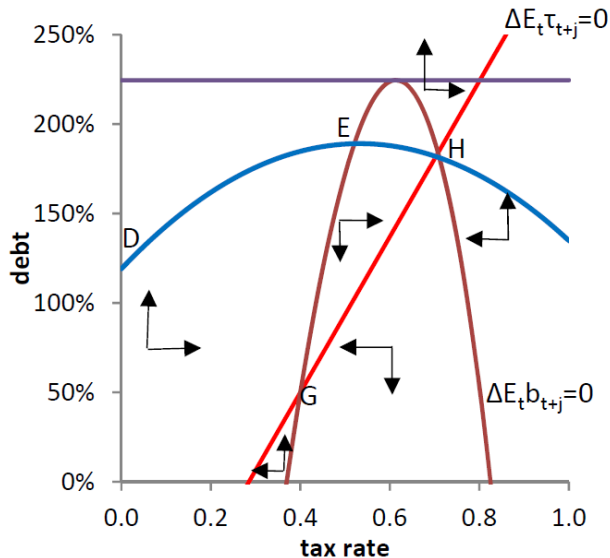


Figure 7: *Estimated effective debt limit*

GDP ratio (z/y) is 0.20,²³ yielding a steady-state debt-to-GDP ratio (b/y) of 0.50 when the discount factor is set to deliver a real interest rate (i) of 4%. The proportionality parameter (λ) is estimated through linear regression of the trade balance over the primary balance, and the estimate for λ is 0.216.²⁴ Using least squares, the estimate for the tax adjustment parameter (γ) is 0.34 and the tax rate persistence (ρ^τ) is 0.63.²⁵ Using an HP filter, we detrend real GDP per worker, real government purchases and real transfer payments to estimate the AR(1) processes for A_t , g_t and z_t . The estimates for the persistence are $\rho^A = 0.44$, $\rho^g = 0.69$ and $\rho^z = 0.31$, and the estimates for the standard deviation are $\sigma^A = 0.015$, $\sigma^g = 0.0011$ and $\sigma^z = 0.0012$. The leisure preference parameter ϕ is set to 2.21 such that the household spends 25% of time working. The total amount of time and the productivity level at the steady-state (A) are normalized to 1.

²³All the variables are from the OECD database (OECD Economic Outlook No. 97). For τ_t we use the total revenue (excluding interest receipts) relative to GDP, for g_t we use the government final current consumption expenditure, and for z_t we use the sum of social security payments and subsidies.

²⁴The standard error is 0.071

²⁵The estimates of the tax rule are reported in the online Appendix.

Our model yields a hump-shaped effective debt limit with values ranging from 122.2% of GDP when the tax rate is close to zero (point D), to 189.6% of GDP when the tax rate is 0.56 (point E), to 183.9% of GDP when the tax rate is 0.71 (point H), as shown in Figure 7.²⁶ Next, we compare our results with Bi's (2012) fiscal limit and Ghosh et al.'s (2013) debt limit. We find that the tax rate at the peak of the Laffer curve (τ_t^L) is 0.614 and Bi's (2012) fiscal limit for Italy (\hat{b}_t^{Bi}) is 224.6% of GDP regardless of the tax rate, as shown by the horizontal line FI in Figure 7.²⁷ The Ghosh et al. (2013) approach cannot produce an estimate for the Italian debt limit.²⁸ Their procedure suggests that the Italian debt has already breached its debt limit, without actually providing an estimate of what the Italian debt limit is.

3.2 Simulations

In this section, we quantify the probability of a solvency crisis. Explicitly, we ask how far is the Italian government debt from its effective debt limit? And what is the probability of a solvency crisis following a series of stochastic shocks? Our model can be solved numerically and simulated to quantify the probability of a crisis over a given period.

To compute the current fiscal space for Italy, we use the parameter values from Table 1, equation (22) and the 2020 values of the tax rate and debt.²⁹ We find that in 2020 the Italian debt was 18.7% of GDP below its effective debt limit, as illustrated in Figure 8. Next, we use the historical values of the tax rate and debt to estimate the Italian fiscal space since

²⁶The debt levels are scaled by the steady-state output (y).

²⁷Bi's procedure yields a distribution. The estimate of 224.6% of GDP can be viewed as the mean of the distribution.

²⁸Their approach requires a somewhat low interest rate to produce an estimate. Our approach does not require the assumptions of Ghosh et al.'s (2013). See the online Appendix for more details on how to compute the Ghosh et al.'s (2013) debt limit.

²⁹ $\tau_{2020} = 0.47$ and $b_{2020} = 181.3\%$ of GDP. Source: OECD Economic Outlook No. 107

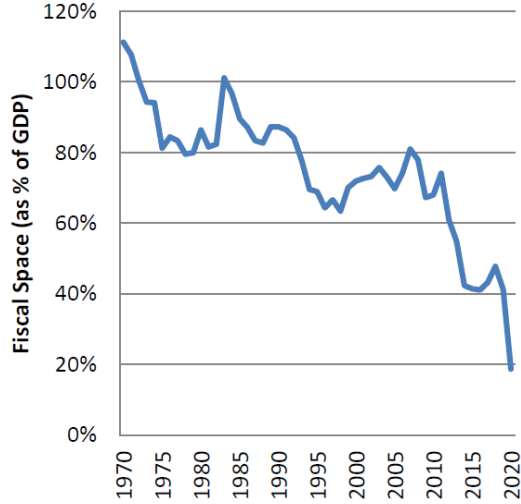


Figure 8: *Fiscal Space*

1970. We find that the fiscal space in Italy has been declining over the last 50 years, and the largest decline of 22.6% of GDP was recorded in 2020. Our results suggest that the global pandemic shock in 2020 had a severe adverse effect on the Italian fiscal space, thus reducing the fiscal space available for the Italian government to maneuver going forward.

Although Italy is below its effective debt limit, it could still experience a solvency crisis because of future adverse shocks. We assume that the shocks $(\varepsilon_t^A, \varepsilon_t^g, \varepsilon_t^z)$ have a normal distribution with mean zero, and we simulate the model to quantify the probability of encountering a solvency crisis over the next ten years.³⁰ Under the baseline parameters values, the probability of a solvency crisis over the next ten years is zero.

Next we estimate the impact of tax austerity on solvency crisis. We consider how the probability of a solvency crisis changes as γ increases beyond its baseline value of 0.34. Figure 9 plots the probability of a solvency crisis as a function of γ . We find that if γ increases by two standard deviations to 0.408, the probability of a crisis remains zero. The crisis probability

³⁰The details of the simulation algorithm are reported in the online Appendix.

becomes positive once γ exceeds 0.413, and unity once γ exceeds 0.444. Our results suggest that the adoption of aggressive tax austerity, captured by increases in the tax adjustment parameter, could push Italy into insolvency. The opposite also holds. A country with a very large γ that eases up on tax austerity could regain access to the markets and re-attain a solvent position.

A very low γ can also lead to problems. We consider how the probability of a solvency crisis changes as γ declines from its baseline value. We repeat the simulations and find that the crisis probability becomes positive once γ falls below 0.124. Our results suggest that whether or not tax austerity could avert or trigger a solvency crisis depends on the country's value of γ . The probability of a crisis does not monotonically decline or rise as γ increases. For very low values of γ , an increase in the tax adjustment parameter lowers the probability of a solvency crisis; however for high values of γ , an increase in the tax adjustment parameter raises the probability of a solvency crisis. For Italy, a γ below 0.124 or above 0.413 could lead to a solvency crisis as shown in Figure 9.³¹

To investigate the sensitivity of the crisis probabilities, we repeat the simulations by changing other values one at a time in the risky direction. Our sensitivity scenarios include: (1) lower initial tax rate (τ_{t-1}), (2) higher persistence in taxes (ρ^τ), and (3) lower proportionality parameter of the trade balance to the primary balance (λ).³² The experiments illustrate three interesting implications of our effective debt limit.

First, the hump-shaped effective debt limit implies that a high-debt country with a low initial tax rate is closer to point D in Figure 7 and is facing a lower value of debt along the

³¹A γ between 0.124 and 0.413 is equivalent to Bohn's (1998) coefficient on debt of between 0.031 and 0.103.

³²See Appendix A for more details about the effects of the persistence (ρ^τ) and the proportionality parameter (λ) on our effective debt limit.

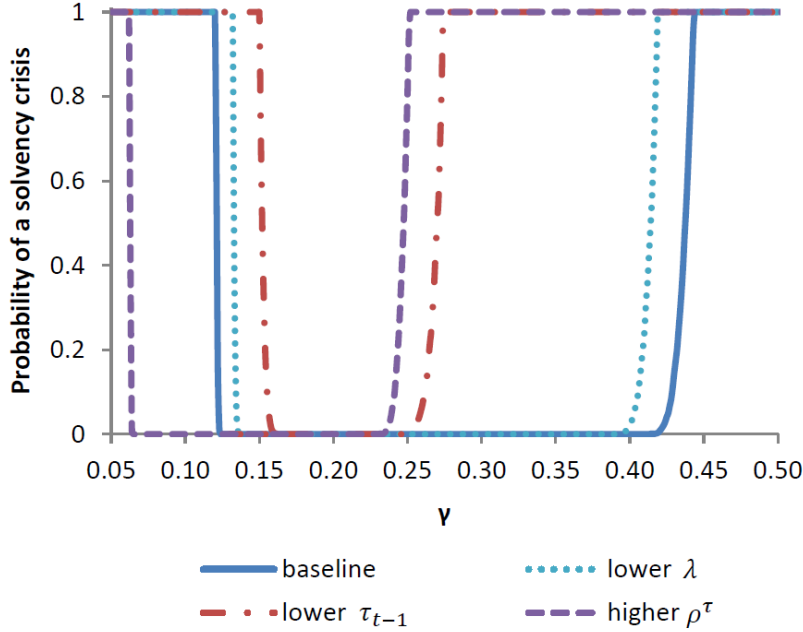


Figure 9: *Probabilities of a solvency crisis as a function of γ*

effective debt limit. When τ_{t-1} is set to the minimum tax rate in our sample of 0.283, the crisis probability is unity at the baseline value of γ . Our results imply that the initial tax rate plays an important role in determining the likelihood of a solvency crisis. If Italy's 2020 value of the tax rate had been substantially lower than 0.47, then the probability of a solvency crisis would have been positive. When we combine the lower initial tax rate ($\tau_{t-1} = 0.283$) with changes in γ , we find that the probability of a crisis becomes positive once γ is below 0.162 or above 0.246. Figure 9 illustrates that the range of values for γ that the probability of a crisis is zero has substantially shrank.

Second, our results imply that a higher persistence in taxes (ρ^τ), most likely stemming from the legislative rigidities or political difficulties in adjusting fiscal policy, substantially increases the crisis probabilities. When ρ^τ increases by two standard deviations from 0.63 to 0.81, the crisis probability rises to unity at the baseline value of γ . The combination of a

higher persistence ($\rho^\tau = 0.81$) with γ reveals that the probability of a crisis is positive once γ is below 0.065 or above 0.230. Our results suggest that a tax rule that exhibits substantial inertia requires a small responsiveness of the tax rate to debt to avoid a solvency crisis. Therefore, high-debt countries with high inertia should refrain from tax austerity.

Third, we find that the decline in the proportionality parameter λ does not substantially affect the probability of a solvency crisis. When λ declines by two standard deviations from 0.216 to 0.074, the probability of a crisis remains zero at the baseline value of γ . A common simplifying assumption in the default literature is that $\lambda = 0$ or $\lambda = 1$. Our results suggest that when the true value of λ is between zero and one, models that assume $\lambda = 0$ overestimate the likelihood of future insolvency, while models that assume $\lambda = 1$ underestimate the likelihood of future insolvency. However, the quantitative effects are modest.

In summary, while the crisis probabilities can be higher under the sensitivity scenarios 1-3 as shown in Figure 9, the implications are identical to the ones under the baseline parameters. High-debt countries with very small γ could lower the probability of a solvency crisis as γ increases, while high-debt countries such as Italy could raise the probability of a solvency crisis as they intensify tax austerity.

Next, we estimate the impact of alternative steady-state fiscal variables (τ, g, z) on the crisis probabilities. The baseline steady-state fiscal variables are calibrated to match the average Italian data between 1970 and 2015, but recently Italy has adopted measures that can be represented by a higher τ or lower g or lower z . We consider the following three cases: the tax rate (τ) increases from 0.40 to 0.41, the government spending-to-GDP ratio (g/y) declines from 0.18 to 0.17, and the transfer payments-to-GDP ratio (z/y) declines

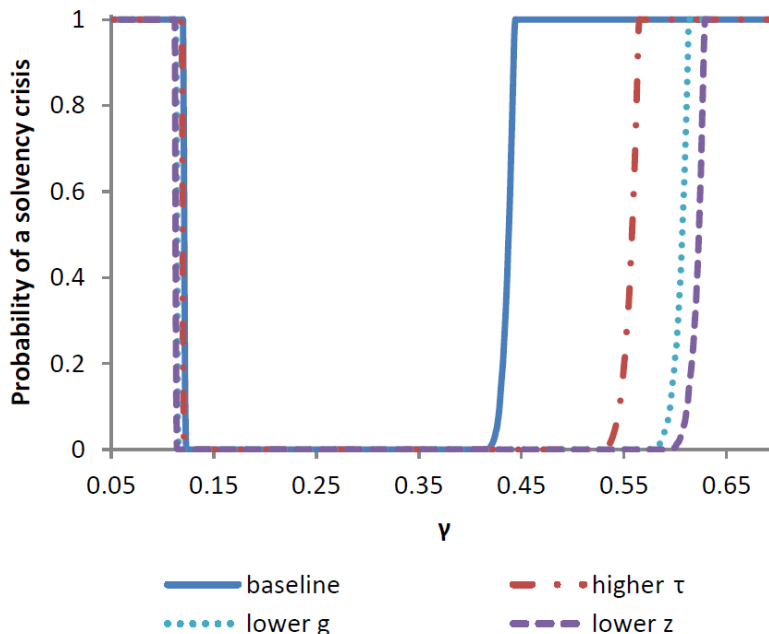


Figure 10: *Probabilities of a solvency crisis as a function of γ*

from 0.20 to 0.19. To illustrate the sensitivity of the crises probabilities to the steady-state values, we repeat the simulations by changing one parameter at a time. Using equation (17), each case yields a higher steady-state primary surplus-to-GDP ratio ($\tau - g/y - z/y$) of 0.03 and a higher steady-state debt-to-GDP ratio (b/y) of 0.75. All three cases make Italy safer, as illustrated in Figure 10. When τ is set to 0.41, the crisis probability becomes positive once γ exceeds 0.524. When g/y is set to 0.17, the crisis probability becomes positive once γ exceeds 0.574. When z/y is set to 0.19, the crisis probability becomes positive once γ exceeds 0.591. Our results imply that high-debt countries that have a higher τ or lower g or lower z raise their effective debt limit and can engage in some tax austerity without backfiring. Additionally, we find that the reduction in transfers has the biggest increase in the range of values for γ that the probability of a solvency crisis is zero.

4 Conclusion

Many countries are implementing strict austerity measures, whereby governments aggressively raise their tax rate as debt rises. In this paper, we show that tax austerity affects the effective debt limit and in turn the likelihood of a solvency crisis. First, we propose an alternative way to derive the effective debt limit. Our approach maps all the fiscal policy parameters to the effective debt limit. Second, we show that tax austerity affects the position of the effective debt limit. We find a nonlinear relationship between tax austerity and the effective debt limit. For small values of the tax adjustment parameter, tax austerity could raise the effective debt limit and prevent a solvency crisis. For larger values of the tax adjustment parameter, tax austerity could lower the effective debt limit and induce a solvency crisis. The latter occurs when the tax rule pushes the tax rate to the wrong side of the Laffer curve. Third, we apply our model to Italy, a high-debt country that is under ongoing pressure from the IMF and EMU to rein in its rising debt level. We estimate the Italian effective debt limit and quantify the impact of intensifying tax austerity on the probability of a solvency crisis. Should Italy implement aggressive tax austerity, our model warns of a potential solvency crisis.

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5 Appendix: Additional properties of the effective debt limit

Our effective debt limit depends on the persistence (ρ^τ) in the tax rule and the proportionality parameter of the trade balance to the primary balance (λ). In this section, we change one parameter at a time and investigate the effects on the effective debt limit.

We begin with the persistence parameter and consider the case where the tax rule, given by equation (6), is not persistent ($\rho^\tau \rightarrow 0$). In Figure 11, a decline in the persistence rotates the $\Delta E_t \tau_{t+j} = 0$ curve counter-clockwise around point G and moves point H westwards ($\frac{\partial \tau^H}{\partial \rho^\tau} > 0$). Persistence is also critical for the hump-shaped specification of our effective debt limit as it affects its curvature. As ρ^τ approaches zero, the effective debt limit becomes horizontal ($\lim_{\rho^\tau \rightarrow 0} \hat{\zeta}_{t-1} \rightarrow 0$) and goes through the new point H. In this case, our effective debt limit is similar, but not identical to Bi's fiscal limit, which is represented by the horizontal line FI. Our approach relaxes the assumption that the peak of the Laffer curve can be maintained forever, and as a result our effective debt limit still depends on the parameter γ .

We then consider the case where the tax rule is highly persistent ($\rho^\tau \rightarrow 1$). An increase in the persistence rotates the $\Delta E_t \tau_{t+j} = 0$ curve clockwise around point G. As ρ^τ goes to unity, the $\Delta E_t \tau_{t+j} = 0$ curve becomes flat and the tax-debt system becomes unstable. The arrows of motion yield adjustment paths which are oscillating away from point G. For the dynamic system to remain locally stable around point G, it requires that the $\Delta E_t \tau_{t+j} = 0$ curve to have a small positive slope. Keeping all other parameters at their baseline values, this requires $\rho^\tau \leq 0.958$. Figure 12 shows that the effective debt limit is substantially lower when the persistence in the tax rule is raised to 0.958, implying that excessive rigidities in

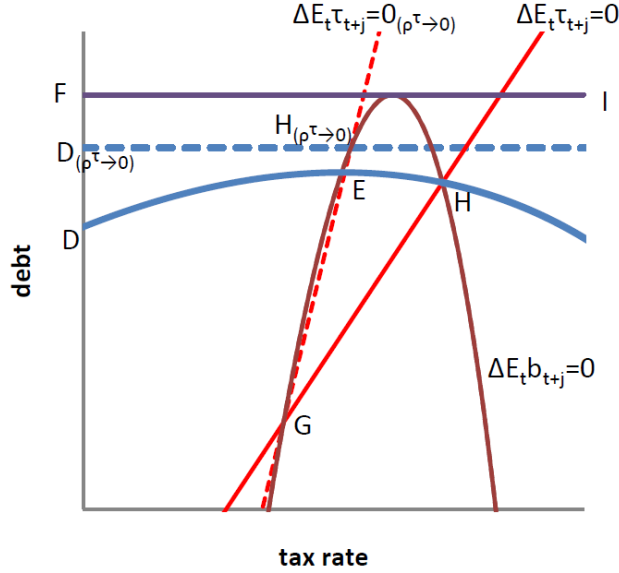


Figure 11: $\rho^\tau \rightarrow 0$

the legislative and implementation process could increase the likelihood of a solvency crisis.

Next, we consider the effects of λ on the effective debt limit. The default literature usually considers two special cases: $\lambda = 1$ and $\lambda = 0$. An increase in λ raises the peak of the $\Delta E_t b_{t+j} = 0$ curve and moves point H in the northeast direction, raising the effective debt limit. Figure 13 shows that our effective debt limit is higher when the trade balance is identical to the primary balance ($\lambda = 1$) because as the primary balance moves into a surplus, the trade balance also improves.

Figure 14 shows the effective debt limit when the trade balance is zero ($\lambda = 0$). A decrease in λ lowers the peak of the $\Delta E_t b_{t+j} = 0$ curve and moves point H in the southwest direction, lowering the effective debt limit. Our framework allows the λ to take any values, including negative ones. A further decrease in λ further lowers the effective debt limit. Figure 15 shows that the effective debt limit when $\lambda = -1$.

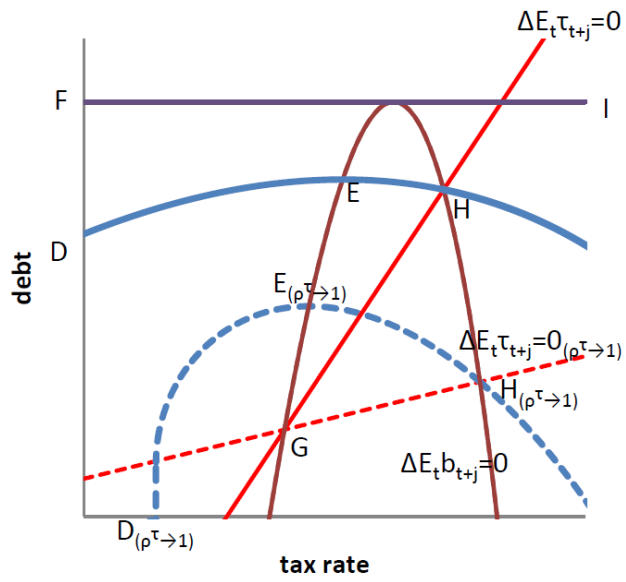


Figure 12: $\rho^\tau \rightarrow 1$

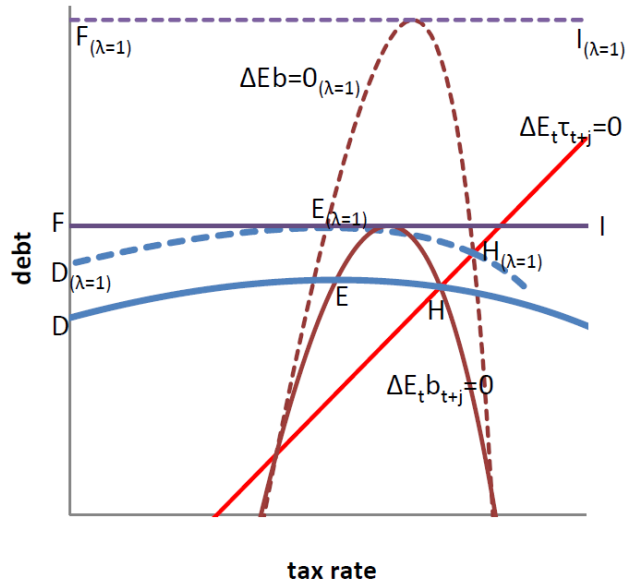


Figure 13: $\lambda = 1$

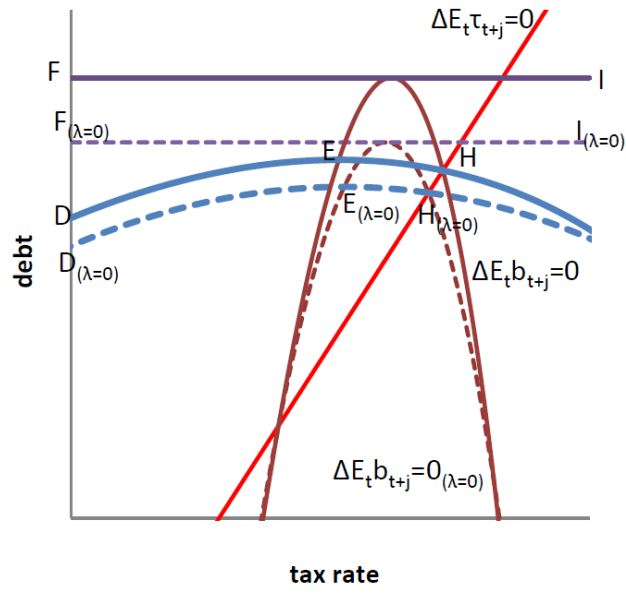


Figure 14: $\lambda = 0$

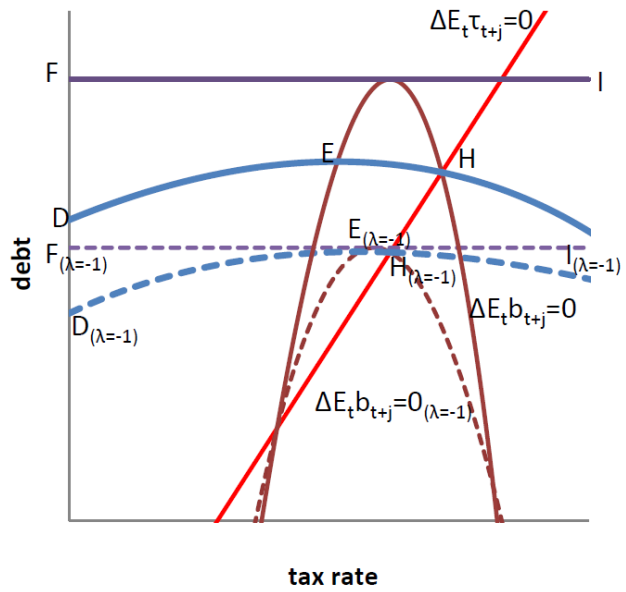


Figure 15: $\lambda = -1$

6 Online Appendix

6.1 Phase Diagram

A phase diagram is an alternative tool to illustrate the dynamics of the model. We use a phase diagram with debt, $E_t b_{t+j-1}$, on the vertical axis and the tax rate, $E_t \tau_{t+j}$, on the horizontal axis. The $\Delta E_t \tau_{t+j} = 0$ curve plots values for debt for each value of the tax rate using equation (15). It is linear in the tax rate and the slope of equation (15) is $\frac{\partial E_t b_{t+j-1}}{\partial E_t \tau_{t+j}} = \frac{1-\rho^\tau}{\gamma} > 0$. The $\Delta E_t b_{t+j} = 0$ curve plots values for debt for each value of the tax rate using equation (16). It yields an inverted U-shaped relationship between the debt and the tax rate. For values of the tax rate less than τ_t^L , the slope of equation (16) is $\frac{\partial E_t b_{t+j-1}}{\partial E_t \tau_{t+j}} > 0$, and for tax rate values larger than τ_t^L the slope of equation (16) is $\frac{\partial E_t b_{t+j-1}}{\partial E_t \tau_{t+j}} < 0$.

The two curves divide the system into different regions, with the arrows of motion revealing the expected direction of movement of the tax rate and debt in each region. For example, in the region that is above the $\Delta E_t \tau_{t+j} = 0$ and $\Delta E_t b_{t+j} = 0$ curves, the arrows of motion point upwards (\uparrow) implying that debt, which is on the vertical axis, is rising and also point to the right (\longrightarrow) implying that the tax rate, which is on the horizontal axis, is also rising. To derive the horizontal arrows of motion, we take the derivative of equation (13) with respect to $E_t \tau_{t+j-1}$, $\frac{\partial \Delta E_t \tau_{t+j}}{\partial E_t \tau_{t+j-1}} = \rho^\tau - 1$, which is negative. Equation (13) is negative to the right of the $\Delta E_t \tau_{t+j} = 0$ curve, yielding the horizontal left arrows for values of the tax rate to the right of $\Delta E_t \tau_{t+j} = 0$ curve. Equation (13) is positive to the left of the $\Delta E_t \tau_{t+j} = 0$ curve, yielding the horizontal right arrows for values of the tax rate to the left of $\Delta E_t \tau_{t+j} = 0$ curve.

To derive the vertical arrows of motion, we take the derivative of equation (14) with

respect to $E_t b_{t+j-1}$, which is positive. Equation (14) is positive above the $\Delta E_t b_{t+j} = 0$ curve, yielding the vertical upward arrows for values of debt above the $\Delta E_t b_{t+j} = 0$ curve. Equation (14) is negative below the $\Delta E_t b_{t+j} = 0$ curve, yielding the vertical downward arrows for values of debt below the $\Delta E_t b_{t+j} = 0$ curve.

6.2 Solvency crisis resolved with default

Here, we derive the expectations of capital loss due to default ($E_{t-1} a_t$). We define a shadow value of capital loss, \tilde{a}_t , which represents the reduction in the value of debt needed for the economy to reach its effective debt limit, equation (21). Setting Ω_t to zero in equation (22) yields

$$\tilde{a}_t = E_{t-1} a_t - x_{t-1} - u_t. \quad (24)$$

Substituting into equation (22) yields an expression for Ω_t as

$$\Omega_t = a_t - \tilde{a}_t.$$

When the shadow value of capital loss via default is positive ($\tilde{a}_t > 0$), default equal to the shadow value sets $\Omega_t = 0$ and restores solvency. When the shadow value is negative, there is no default.

To solve for the magnitude of default, a_t , we must first solve for the expectations of default, $E_{t-1} a_t$. Define u_t^* as a critical value for the aggregate shock u_t such that

$$\begin{aligned} a_t &> 0 \text{ for } u_t < u_t^* \\ a_t &= 0 \text{ for } u_t \geq u_t^*. \end{aligned}$$

Letting $f(u_t)$ be a bounded, symmetric, mean-zero distribution for u_t , with bounds given by $\pm \bar{u}$, the probability of a solvency crisis can be expressed as

$$F(u_t^*) = \int_{-\bar{u}}^{u_t^*} u_t f(u_t)$$

and the expectation for equation (24) can be written as

$$E_{t-1}a_t = \int_{-\bar{u}}^{u_t^*} a_t f(u_t) = \int_{-\bar{u}}^{u_t^*} (E_{t-1}a_t - x_{t-1} - u_t) f(u_t).$$

Collecting terms on the expectation of default yields

$$[1 - F(u_t^*)] E_{t-1}a_t = -x_{t-1}F(u_t^*) - \int_{-\bar{u}}^{u_t^*} u_t f(u_t). \quad (25)$$

Substituting into equation (24), yields an implicit expression for a_t

$$[1 - F(u_t^*)] a_t = - \left[x_{t-1} + u_t (1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) \right]. \quad (26)$$

There is a solution for u_t^* iff $x_{t-1} \geq 0$. For large positive values of x_{t-1} , the critical value of the shock (u_t^*) equals its lower bound ($-\bar{u}$). As x_{t-1} falls, u_t^* rises, reaching its upper bound at \bar{u} once $x_{t-1} = 0$. For negative values of x_{t-1} , even the upper support (\bar{u}) does not satisfy the equation (26) because when $u_t^* = \bar{u}$, the left-hand side of the equation (26) is zero and the right-hand side is positive. Therefore, existence of an equilibrium value for expected default requires that $x_{t-1} \geq 0$.

6.2.1 Contacts with the literature

In this section we compare our effective debt limit with the limits proposed by Bi (2012) and Ghosh et al. (2013). Bi's (2012) fiscal limit, which we label as \hat{b}_t^{Bi} , is the expected present value of the future maximum primary surpluses that the government can raise, and can be written as

$$\hat{b}_t^{Bi} = E_t \sum_{k=1}^{\infty} (\tau_t^L y_{t+k} - g_{t+k} - z_{t+k}) \left(\prod_{j=1}^k \frac{1}{1 + i_{t+j}} \right). \quad (27)$$

It represents the highest borrowing a country can have and is associated to the tax rate at the peak of the Laffer curve (τ_t^L), given by equation (12). Her procedure, however, assumes

that a government can instantaneously raise its current tax rate to τ_t^L and maintain it at τ_t^L forever. Therefore, equations (12) and (27) are independent of τ_t and γ . In Figures 4 and 5, Bi's (2012) fiscal limit is represented by the horizontal line FI, which is tangent to the peak of the $\Delta E_t b_{t+j} = 0$ curve, implying that Bi's fiscal limit is the same, regardless of τ_t and γ .³³ But, what if the government is unable to raise its tax rate immediately to the peak of the Laffer curve and maintain it forever? This paper extends her paper in two key ways.

First, we relax the assumption that the peak of the Laffer curve can be attained instantaneously. We introduce persistence in the tax rule to capture the inertia in adjusting the tax rate. The inclusion of persistence (ρ^τ) in the tax rate is critical for the hump-shaped specification. As ρ^τ approaches zero, the effective debt limit becomes horizontal ($\lim_{\rho^\tau \rightarrow 0} \hat{\zeta}_{t-1} \rightarrow 0$) and goes through point H.³⁴ Second, we relax the assumption that τ_t^L can be maintained indefinitely. We assume that the government follows its tax rule in which the tax rate responds systematically to the debt level. When a government cannot maintain its tax rate at τ_t^L forever, we obtain an effective debt limit that depends on all the fiscal policy parameters, including γ . Additionally, after we relax both assumptions, we find that the peak of our effective debt limit (point E) does not necessarily occur at τ_t^L . Figure 3 shows an effective debt limit whose highest position (point E) occurs when the tax rate is less than τ_t^L , implying that a country does not have to raise its tax rate to τ_t^L to reach the maximum value of debt consistent with solvency.

The Ghosh et al. (2013) approach is based on the stability properties of a reduced form cu-

³³It is important to note that the phase diagram illustrates both Bi's fiscal limit and our effective debt limit when shocks are equal to their expected values of zero. Bi (2012) has shown that shocks yield a distribution for her fiscal limit along the horizontal line FI. When we allow the shocks to take nonzero values, our effective debt limit also has a distribution around the DEH curve.

³⁴For more details see the Appendix in the paper.

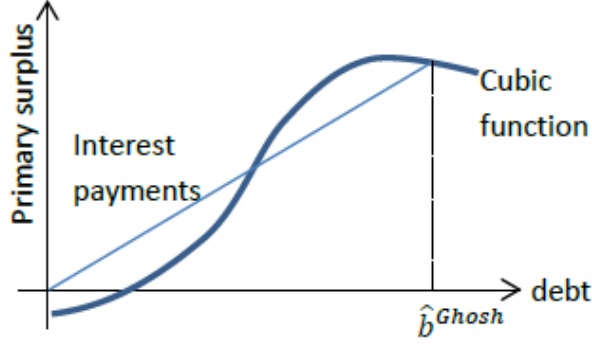


Figure 16: *Debt limit of Ghosh et al. (2013)*

bic fiscal reaction function that governs the evolution of the primary surplus ($s_t = \tau_t y_t - g_t - z_t$), and is given by

$$s_t = c + \gamma_1 b_{t-1} + \gamma_2 b_{t-1}^2 + \gamma_3 b_{t-1}^3 + \varepsilon_t^s, \quad (28)$$

where ε_t^s represents the primary surplus shocks. A negative coefficient on the cubic debt term ($\gamma_3 < 0$) implies that the primary surplus weakens as debt increases, a phenomenon that the authors termed as fiscal fatigue, and eventually there is a point beyond which the primary surplus is not sufficient to pay the interest on debt. At that point, the dynamic system becomes unstable. When debt enters this unstable region, it becomes explosive and agents refuse to lend, creating a solvency crisis. Ghosh et al. (2013) argue that the point at the boundary of the unstable region represents the debt limit, which we label as \hat{b}^{Ghosh} , and is given by the largest root of the following equation

$$c + \gamma_1 b_{t-1} + \gamma_2 b_{t-1}^2 + \gamma_3 b_{t-1}^3 + \varepsilon_t^s = i b_{t-1} \quad (29)$$

which equates the cubic function of the primary surplus with interest payments ($i b_{t-1}$), and is illustrated in Figure 16.

The Ghosh et al. (2013) procedure, however, relies on two crucial assumptions. First, it requires a nonlinear fiscal reaction function with $\gamma_3 < 0$ to capture fiscal fatigue. If either

γ_3 is positive or the fiscal reaction function is linear ($\gamma_2 = \gamma_3 = 0$), then the Ghosh et al. (2013) approach cannot identify a debt limit. Second, even if the fiscal reaction function is nonlinear with $\gamma_3 < 0$, the estimation of the debt limit in equation (29) requires a somewhat low interest rate (i). A very high i rotates the interest payment curve counterclockwise around the origin such that equation (29) has no solution.³⁵ Our approach does not require the assumptions of Ghosh et al. (2013).

Fiscal fatigue is an empirical feature identified by Ghosh et al. (2013). We use our framework to provide a theoretical justification, thereby bridging the gap between the theoretical and empirical work on fiscal fatigue and debt limits. We show that a tax rule in conjunction with distortionary taxes can limit the tax revenue that the government generates as the tax rate moves beyond the peak of the Laffer curve, and in turn gives rise to the fiscal fatigue phenomenon. Fiscal fatigue requires the primary surplus responsiveness to debt to weaken as debt rises. In a cubic regression, a negative coefficient on the cubic debt term yields fiscal fatigue. In a quadratic regression, a negative coefficient on the squared debt term is sufficient for fiscal fatigue (Shiamptanis 2015). Using equations (6) and (11), the primary surplus ($s_t = \tau_t y_t - g_t - z_t$) can be written as a quadratic function in terms of debt

$$\begin{aligned}
 s_t = & \underbrace{\frac{A_t ((1 - \rho^\tau) \tau - \gamma b + \rho^\tau \tau_{t-1}) (1 - ((1 - \rho^\tau) \tau - \gamma b + \rho^\tau \tau_{t-1})) + \phi (1 - \lambda) g_t - \phi \lambda z_t}{1 + \phi - (1 + \phi \lambda) \tau_t}}_{\text{intercept}} - (g_t + z_t) \\
 & + \underbrace{\frac{A_t \gamma (1 - 2 ((1 - \rho^\tau) \tau - \gamma b + \rho^\tau \tau_{t-1}))}{1 + \phi - (1 + \phi \lambda) \tau_t}}_{\text{coefficient on linear debt}} b_{t-1} \\
 & - \underbrace{\frac{A_t \gamma^2}{1 + \phi - (1 + \phi \lambda) \tau_t}}_{\text{coefficient on squared debt}} b_{t-1}^2.
 \end{aligned} \tag{30}$$

The quadratic equation relates the primary surplus to debt with a negative coefficient on

³⁵Robertson and Tambakis (2016) identify additional cases where the Ghosh et al. (2013) approach cannot provide information about the debt limit, even though the fiscal reaction function is nonlinear with $\gamma_3 < 0$.

the squared debt term. We derive the reduced form coefficients of a quadratic regression in terms of the fundamental parameters of our model, and we find that the magnitude of the negative coefficient on squared debt, in equation (30), increases as γ increases or τ_t rises, implying that tax austerity enhances fiscal fatigue.

To summarize, our effective debt limit extends both Bi's (2012) and Ghosh et al. (2013) as it maps all the fiscal policy parameters to the limit and it endogenizes the fiscal fatigue phenomenon.

6.3 Estimates of fiscal rules and Ghosh et al. (2013) debt limit

We estimate equation (6) using least squares and annual data from 1970 to 2015. All the variables are from the OECD database (OECD Economic Outlook No. 97). For the tax rate (τ_t) we use the total revenue (excluding interest receipts) relative to GDP, for the debt level (b_t) we use the general government gross financial liabilities, and for *output gap* we use the economy's output gap. The estimates of γ from Regressions (1) and (2) in Table 2 are equivalent to the Bohn's (1998) coefficient on debt-to-GDP of 0.0849 and 0.0715, respectively.

	(1)	(2)
ρ	0.6344*** (0.0882)	0.7056*** (0.0870)
γ	0.3396*** (0.0336)	0.2864*** (0.0284)
<i>output gap</i>	-0.1050 (0.0797)	
\bar{R}^2	0.7539	0.6911

Note: The *, ** and *** indicate statistical significance at the 90, 95 and 99 percent level, respectively.

To compute Ghosh et al. (2013) debt limit, we first estimate equation (28) using least

squares and annual data from 1970 to 2015. The estimates are presented in Table 3.³⁶ The coefficient on the cubic term is negative, but enters significantly at the 99 percent level when lagged primary surplus is excluded.³⁷ To estimate \hat{b}^{Ghosh} we use the fiscal policy parameters from Regression 1 in Table 3, and the average growth-adjusted interest rate over the last ten years as in Ghosh et al. (2013).³⁸ Similar to their results, equation (29) has no solution for Italy. We cannot obtain an estimate for the Italian debt limit using their approach because the interest payment curve is always above the estimated cubic function, as shown in Figure 17. The Ghosh et al. (2013) procedure suggests that the Italian debt has already breached its debt limit and is on an explosive path, without actually providing an estimate of what the Italian debt limit is.

Table 3: Estimates of the cubic fiscal rule

	(1)	(2)
c	36.8213*** (9.7867)	10.8086 (10.2374)
b_{t-1}	-1.4857*** (0.3025)	-0.5327 (0.3386)
b_{t-1}^2	0.0163*** (0.0030)	0.0063* (0.0030)
b_{t-1}^3	-0.00005*** (0.000009)	-0.00002* (0.00001)
<i>output gap</i>	0.2175** (0.0030)	0.1591* (0.0838)
<i>spending gap</i>	-0.0884 (0.0626)	-0.0758 (0.0525)
s_{t-1}		0.4969*** (0.1171)
\bar{R}^2	0.8356	0.8847

Note: The *, ** and *** indicate statistical significance at the 90, 95 and 99 percent level, respec-

³⁶All the variables are from the OECD database (OECD Economic Outlook No. 97). For s_t we use the general government primary balances relative to GDP, for b_t we use the general government gross financial liabilities relative to GDP, for *output gap* we use the economy's output gap, and for *spending gap* we use the cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter.

³⁷Lagged primary surplus captures persistence.

³⁸In Italy, the average growth-adjusted interest rate between 2005 and 2015 is 3%.

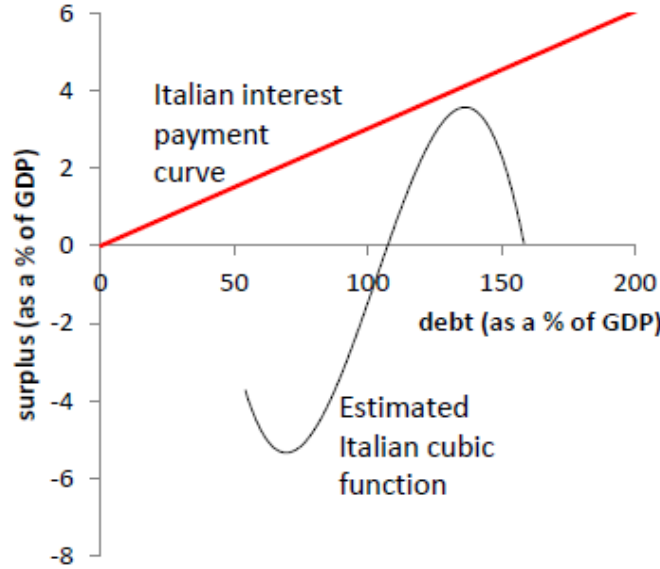


Figure 17: *Estimated cubic function*

tively.

6.4 Simulation algorithm

Table 4: Calculating the probability of a solvency crisis over the next ten years

1. Compute the state variable determining the fiscal space, x_{t-1} , from eq. (23) using initial values of debt, b_{t-1} , tax rate, τ_{t-1} , government spending, g_{t-1} , transfers, z_{t-1} , and technology, A_{t-1} .
2. Compute the expectations of capital loss due to default, $E_{t-1}a_t$, from eq. (25).
3. Compute the interest rate, i_{t-1} , and the expectations of default, $E_{t-1}\delta_t$, from eq. (2) and $E_{t-1}a_t = (1 - E_{t-1}\delta_t)(1 + i_{t-1})b_{t-1}$.
4. Draw a productivity shock, ε_t^A , a government spending shock, ε_t^g , and a transfer shock, ε_t^z from $N(0, \sigma^A)$, $N(0, \sigma^g)$ and $N(0, \sigma^z)$, respectively.
5. Calculate the value for capital loss due to default, a_t , from eq. (26).
6. If $a_t > 0$, then there is a solvency crisis with $\delta_t < 1$, and the simulation ends.
7. If $a_t = 0$, then $\delta_t = 1$ and next period's debt, taxes, government spending, transfers and productivity are updated which are then used to update x_t .
8. Repeat steps 2-7 for ten years.
9. Repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

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