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## Risk Assessment Under a Nonlinear Fiscal Policy Rule

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## Abstract

In the aftermath of the recent debt crisis, many countries are implementing nonlinear fiscal policy rules, whereby the government's responsiveness to debt must strengthen at higher levels of debt. This paper examines how a nonlinear fiscal policy rule affects the possibility of future insolvency in a small open economy. We find that (1) the criteria for a nonlinear fiscal rule to eliminate explosive behavior should be tighter than the ones proposed by Bohn (1998); (2) a country that adopts a nonlinear fiscal rule could substantially reduce the probability of a solvency crisis; (3) a nonlinear fiscal rule allows a country to reduce the possibility of insolvency without large initial responsiveness.

- *Key Words:* Nonlinear fiscal rule, Fiscal Sustainability, Solvency Crisis, Policy Switching, Canada
- *JEL Classification:* C63, E62, E63, F34, H63

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# 1 Introduction

The recent debt crisis highlighted the importance of designing better fiscal rules for the future. Countries around the world are establishing nonlinear fiscal rules, whereby governments must respond more aggressively to debt when debt is above some threshold level. In Europe, many countries are introducing a German-style “debt break” into their constitutions. Outside Europe, governments are adopting fiscal rules by which authorities have to strengthen their responsiveness at high levels of debt. Schaechter et al. (2012) find that the number of countries around the world with fiscal rules spiked to 72 in 2012. Nonlinear fiscal rules allow for an endogenous increasing response to rising debt and capture some of the policy ideas currently being considered or implemented in many countries with the hope to allay future solvency crisis. The purpose of this paper is to study the implications of a nonlinear fiscal rule on solvency crisis. Do nonlinear fiscal rules affect the possibility of future insolvency? Does the increased responsiveness suggested by the nonlinear fiscal rules need to be exhibited at low levels of debt? Does the response below the threshold level matter? This paper addresses these issues.

Bohn (1998) was the first to explore theoretically the nonlinear relationship between the primary surplus and debt, although it was not the thrust of his paper. He utilizes a fiscal reaction function that describes the evolution of the primary surplus and argues that any positive marginal response of the primary surplus to debt satisfies the government’s intertemporal budget constraint (IBC) and thus assures fiscal sustainability. However, Bohn’s analysis does not incorporate a fiscal limit on the size of debt, defined as the maximum level

of debt that the country can repay (Bi 2012; Bi et al. 2010, 2013; Cochrane 2011; Ghosh et al. 2013). Although a small positive marginal response of the primary surplus to debt satisfies the IBC, it allows debt relative to GDP to grow forever and eventually violates any fiscal limit on debt. This paper can be viewed as an extension of Bohn (1998) when a country faces fiscal limits. We combine the nonlinear responsiveness to debt and the fiscal limits to examine the effects on solvency.

We specify a simple nonlinear fiscal rule that governs the evolution of the primary surplus relative to GDP and derive the criteria necessary for the nonlinear rule to satisfy the government's IBC and also eliminate explosive behavior. We find that for all values of debt the marginal response of the primary surplus to debt should be larger than the interest rate times an adjustment factor for the persistence in the primary surplus, which is larger than Bohn's criterion. However, a fiscal rule which eliminates explosiveness is not sufficient to assure the absence of a solvency crisis.

Our framework builds on and extends the setup developed by Daniel and Shiamptanis (2012). A government following a nonlinear fiscal rule could receive negative shocks, such as the 2007-2009 worldwide financial turmoil, sending it to a position where agents refuse to lend, creating a solvency crisis. Monetary and fiscal authorities need a policy response to restore lending. They could agree to implement a policy-switch in which the fiscal policy switches to active and the monetary policy to passive.<sup>1</sup> The switch usually requires debt devaluation via surprise inflation to reduce debt.<sup>2</sup>

We apply the model to Canada, a country which has shown that it responds nonlinearly

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<sup>1</sup> Following Leeper's (1991) terminology, an active policy is free to pursue its objectives, while a passive authority responds to debt and is constrained by the active authority's actions.

<sup>2</sup> Davig et al. (2010, 2011) present dynamic policy-switching models in which the fiscal authority follows linear policy rules and switching between the two regimes occurs exogenously

to the state of government indebtedness, adjusting more aggressively when the debt to GDP ratio is above a certain threshold level. We estimate a nonlinear fiscal rule with annual data from 1970-2012 and then utilize it to quantify the probability of solvency crisis.

The following results emerge. First, a nonlinear fiscal rule naturally yields an endogenous stochastic marginal response of the primary surplus to debt, without the need for considering stochastic switching between two linear fiscal rules to capture changes over time in policy. Second, a nonlinear fiscal rule lowers the expected maximum level of debt on the path back to its long-run target and also shortens the expected adjustment time. Third, the strong responsiveness suggested by a nonlinear fiscal rule is very important to reduce the probability of a solvency crisis, however this strength need not be exhibited until debt is high.

This paper is organized as follows. Section 2 derives the necessary conditions for global stability, and the dynamics leading to a solvency crisis under a nonlinear fiscal rule. Section 3 estimates the policy parameters of the Canadian fiscal rule and the probability of a solvency crisis. Section 4 provides conclusions.

## 2 Model

We set up a simple open economy model with a stochastic endowment and a nonlinear fiscal policy rule, which we use to address the probability of a solvency crisis. Initially, monetary policy is active and fiscal policy is passive (as in Leeper 1991). We assume a two-country, two-currency, one good world. The domestic country is small enough that it cannot affect the foreign price level and foreign interest rate. With a single good in the world, goods market equilibrium requires the law of one price. Normalizing the foreign price level at unity ( $P_t^f = 1$ ) and assuming no foreign inflation implies that the equilibrium domestic price level

is the exchange rate.

## 2.1 Agent

Our domestic small open economy is populated by a representative agent, who maximizes the following utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where  $0 < \beta < 1$  is the discount factor,  $E_t$  denotes the expectation conditional on the information at time  $t$ , and  $c_t$  is real consumption, subject to the budget constraint

$$\frac{B_t^{h,d}}{P_t} + B_t^{h,f} + c_t = (1 + i_{t-1}) \frac{B_{t-1}^{h,d}}{P_t} + (1 + i) B_{t-1}^{h,f} + (1 - \tau_t) y_t,$$

where  $B_t^{h,d}/P_t$  and  $B_t^{h,f}$  denote the real value of bonds held by the home agent in domestic-currency and foreign-currency, respectively,  $P_t$  denotes the domestic price level,  $i_{t-1}$  is the interest rate that the domestic-currency bond pays,  $i$  is the risk-free interest rate that the foreign-currency bond pays, which is assumed to be constant,  $y_t$  is real output, and  $\tau_t$  is the tax rate.

In the event of a solvency crisis, the domestic government adopts policy switching, which usually requires debt devaluation via inflation. We assume that agents know the policy response to the crisis. Therefore, domestic-currency bonds are riskier assets than foreign-currency bonds. The first-order conditions are

$$\beta E_t (1 + i_t) \frac{P_t}{P_{t+1}} u_c(c_{t+1}) = u_c(c_t) \quad \text{and}$$

$$\beta E_t (1 + i) u_c(c_{t+1}) = u_c(c_t).$$

The foreign agent faces an analogous budget constraint and Euler equations. We assume that the foreign agent is willing to buy the domestic-currency bonds as long as their interest

rate,  $i_t$ , satisfies interest rate parity. Interest rate parity is derived from the Euler equations when the covariance between the domestic interest rate and the foreign agent's consumption is zero<sup>3</sup>, and it can be expressed as

$$\frac{1}{1+i_t} = \frac{1}{1+i} E_t \frac{1}{1+\pi_{t+1}}. \quad (1)$$

where  $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$  is the domestic inflation rate. Equation (1) implies that the domestic country's interest rate,  $i_t$ , rises above the foreign interest rate,  $i$ , when there is some possibility of a solvency crisis which will be resolved with debt devaluation through inflation,  $E_t \frac{1}{1+\pi_{t+1}} < 1$ .

## 2.2 Government flow budget constraint

The government in a small open economy can issue interest-bearing debt denominated in both domestic and foreign currency. The domestic government's real flow budget constraint is given by

$$\frac{B_t}{P_t} + B_t^f = (1+i_{t-1}) \frac{B_{t-1}}{P_t} + (1+i) B_{t-1}^f + g_t - \tau_t y_t.$$

where  $B_t/P_t$  and  $B_t^f$  denote the real value of bonds issued by the domestic government in domestic and foreign currency, respectively,  $g_t$  is real government expenditures, and  $\tau_t y_t$  is real tax revenue. Dividing by real output,  $y_t$ , domestic-currency debt relative to output,  $b_t$ , foreign-currency debt relative to output,  $b_t^f$ , and the primary surplus relative to output,  $s_t$ , can be expressed respectively as

$$b_t = \frac{B_t}{P_t y_t},$$

$$b_t^f = \frac{B_t^f}{y_t},$$

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<sup>3</sup> This follows from the small open economy assumption.

$$s_t = \frac{\tau_t y_t - g_t}{y_t}.$$

The domestic government's flow budget constraint can be expressed in terms of debt and the primary surplus relative to output as

$$b_t + b_t^f = \left( \frac{1 + i_{t-1}}{(1 + \pi_t)(1 + \rho_t)} \right) b_{t-1} + \left( \frac{1 + i}{1 + \rho_t} \right) b_{t-1}^f - s_t$$

where  $\rho_t = \frac{y_t}{y_{t-1}} - 1$  is the stochastic real output growth rate. Imposing interest rate parity from equation (1) and rearranging yields

$$b_t + b_t^f = \left( \frac{1 + i}{1 + \rho_t} \right) (b_{t-1} + b_{t-1}^f) - s_t - \left( E_{t-1} \frac{1}{1 + \pi_t} - \frac{1}{1 + \pi_t} \right) \left( \frac{1 + i_{t-1}}{1 + \rho_t} \right) b_{t-1}.$$

Define  $\gamma_t$  as devaluation on domestic-currency debt due to inflation as

$$\gamma_t = \left( 1 - \frac{1}{1 + \pi_t} \right) \left( \frac{1 + i_{t-1}}{1 + \rho_t} \right) b_{t-1}, \quad (2)$$

where inflation,  $\pi_t > 0$ , reduces the value of outstanding domestic-currency debt,  $\gamma_t > 0$ , and when  $\pi_t = 0$ , devaluation does not occur,  $\gamma_t = 0$ . Using (2), the equation for the evolution of total debt relative to output,  $d_t = b_t + b_t^f$ , can be expressed as

$$d_t = (1 + r_t) d_{t-1} - s_t - (\gamma_t - E_{t-1} \gamma_t) \quad (3)$$

where  $r_t = \left( \frac{1+i}{1+\rho_t} \right) - 1$  is the risk-free growth-adjusted interest rate and  $(\gamma_t - E_{t-1} \gamma_t)$  represents the unexpected inflation or, equivalently, a price level shock, which reduces the value of domestic-currency debt and contributes to government revenue.<sup>4</sup>

Using equation (3) together with the assumption that a government does not allow its debt to become negative in the limit<sup>5</sup>, the domestic government's intertemporal budget

<sup>4</sup> If a government issues a larger fraction of its bonds in foreign-currency than in domestic-currency,  $(\gamma_t - E_{t-1} \gamma_t)$  might not provide the necessary revenue to reduce debt.

<sup>5</sup> Sims (1997), Woodford (1997), and Daniel (2001) argue that no country, acting to maximize utility of its own agents, would allow the present-value of its debt to become negative in the limit.



constraint (IBC) is given by

$$\lim_{T \rightarrow \infty} E_t d_{t+T} \left( \prod_{i=1}^T \frac{1}{1+r_{t+i}} \right) = d_t - E_t \sum_{k=1}^{\infty} s_{t+k} \left( \prod_{i=1}^k \frac{1}{1+r_{t+i}} \right) = 0. \quad (4)$$

### 2.3 Policy rules

The monetary authority is free to determine inflation with an active monetary policy. We assume that the active monetary policy sets the domestic interest rate,  $i_t$ , according to the following Taylor rule

$$i_t = i + \kappa (\pi_t - \pi) \quad \kappa > 1, \quad (5)$$

where initial inflation and inflation target,  $\pi$ , is set to zero.

We assume that the fiscal authority follows a simple nonlinear passive fiscal rule that governs the evolution of primary surplus relative to GDP. A nonlinear responsiveness of the primary surplus to debt was found in the U.S. by Bohn (1998, 2008), and Greiner and Kauermann (2007), and in a panel of 23 advanced economies by Ghosh et al. (2013). To explore the nonlinear relationship between the primary surplus and debt, we generalize the fiscal rule of Bohn (1998) by adding a spline term of the form  $\max(0, d_{t-1} - d')$ <sup>2</sup>, which picks out periods with debt above its threshold level,  $d'$ .<sup>6</sup> The primary surplus relative to GDP responds to a constant, its own lag, lagged debt relative to GDP, output gap,  $\tilde{y}_t$ , and when debt is above  $d'$ , it also responds to square deviations of lagged debt from its threshold level, and is given by

$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max(0, d_{t-1} - d')^2 + \beta_4 \tilde{y}_t + \nu_t, \quad (6)$$

where  $\tilde{y}_t$  is the percentage deviation of real output from its potential level, and  $\nu_t$  represents the bounded, zero-mean stochastic fiscal shocks. Output gap,  $\tilde{y}_t$ , captures the "automatic

<sup>6</sup> We show in the empirical section that this term is statistically significant.

response" or non-discretionary response to business cycle fluctuations. Fiscal shocks,  $\nu_t$ , are random and represent both truly unanticipated fiscal shocks, as with a war, natural disaster, commodity price fluctuations, or bank bailouts, as well as discretionary fiscal policy responses to the state of the economy. Most recently, they reflect the fiscal response to the global financial crisis of 2007-2009.

The spline term applies when debt exceeds its threshold level,  $d'$ . This yields a nonlinear fiscal rule in which the government increases its primary surplus responsiveness to debt as debt rises above its threshold value. An important feature of the nonlinear fiscal rule is that the marginal response of the primary surplus to debt,  $\frac{\partial s_t}{\partial d_{t-1}} = \beta_2 + 2\beta_3 \max(d_{t-1} - d')$ , for  $d_{t-1} > d'$  is time-varying, with values depending on the realization of the debt level. The time-varying marginal response naturally captures the changes in fiscal policy over time at high levels of debt.<sup>7</sup>

## 2.4 Output

To complete the model, we specify output dynamics as

$$y_t = (1 + \rho) y_{t-1} + \varepsilon_t y_{t-1}, \quad (7)$$

where  $\rho$  is the long-run growth rate of output, and  $\varepsilon_t$  is a mean-zero productivity shock. Our assumption that real output is an independent stochastic process is consistent with a model in which output is driven by exogenous productivity shocks. Following Bohn (1998, 2008),

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<sup>7</sup> Additionally, the spline is necessary to render stability at low levels of debt. Alternatively, if the spline term is removed and the square term applies for all values of  $d_{t-1}$  as in Bohn (1998), then the marginal response of the primary surplus to debt,  $\frac{\partial s_t}{\partial d_{t-1}} = \beta_2 + 2\beta_3 (d_{t-1} - d')$ , will be negative for values of debt substantially below the threshold level. This suggests that when debt is very low, a government will increase its primary surplus in response to a fall in debt. This is an unstable area because it will further decrease debt. This instability and unorthodox property for low values of debt is ruled out with the use of the spline term. Bohn (2008) also noted that nonlinearities could raise concerns about the stability of fiscal policy at low levels of debt. However, he did not explicitly address this issue.

Ghosh et al. (2013) and Leeper (2010), we are assuming that the primary surplus has no impact on output.<sup>8</sup> Our specification should be viewed as a simplification, which is common in the literature on fiscal sustainability.

The productivity shocks provide an additional source of uncertainty aside from fiscal shocks. Using equation (7), the stochastic real output growth rate,  $\rho_t$ , and the growth-adjusted interest rate,  $r_t$ , can be expressed as

$$\begin{aligned}\rho_t &= \rho + \varepsilon_t, \\ r_t &= r - \frac{(1+r)\varepsilon_t}{1+\rho_t},\end{aligned}\tag{8}$$

where  $r = \left(\frac{1+i}{1+\rho}\right) - 1$  is the average long-run growth-adjusted interest rate. Define  $\varepsilon_t^d$  as the impact of productivity shocks on debt as

$$\varepsilon_t^d = - \left( \frac{(1+r)\varepsilon_t}{1+\rho_t} \right) d_{t-1},\tag{9}$$

and  $\varepsilon_t^s$  as the impact of productivity shocks on the primary surplus as

$$\varepsilon_t^s = \frac{\beta_4 \varepsilon_t y_{t-1}}{y_t^p},\tag{10}$$

where  $y_t^p = (1+\rho)y_{t-1}^p$  is the potential real GDP. A negative productivity shock ( $\varepsilon_t < 0$ ) reduces the primary surplus ( $\varepsilon_t^s < 0$ ). Additionally, it increases the growth-adjusted interest rate above its long-run rate ( $r_t > r$ ), and accelerates the rate at which debt relative to GDP accumulates ( $\varepsilon_t^d > 0$ ). If  $\varepsilon_t = 0$ , then  $r_t = r$ ,  $\varepsilon_t^s = 0$  and  $\varepsilon_t^d = 0$ .

Using equations (6), (7) and (10), the primary surplus can be written as<sup>9</sup>

$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max\left(0, d_{t-1} - d'\right)^2 + \beta_4 \tilde{y}_{t-1} + \varepsilon_t^s + \nu_t.\tag{11}$$

<sup>8</sup> When we add real primary surplus in equation (7), we find that its coefficient is not statistically different from zero. In the macro literature, the sign of this effect is controversial and model specific.

<sup>9</sup> Substituting equation (7) into the output gap,  $\tilde{y}_t = \frac{y_t - y_t^p}{y_t^p}$ , the output gap can be expressed as  $\tilde{y}_t = \tilde{y}_{t-1} + \frac{\varepsilon_t y_{t-1}}{y_t^p}$ .

Substituting equation (8) into equation (3) and using equation (9), the evolution of debt relative to output can be written as

$$d_t = (1 + r) d_{t-1} - s_t - \gamma_t + E_{t-1}\gamma_t + \varepsilon_t^d. \quad (12)$$

Equation (12) allows us to isolate the terms that increase the interest rate. The expectations of devaluation via inflation,  $E_{t-1}\gamma_t > 0$ , and negative productivity shocks,  $\varepsilon_t^d > 0$ , raise the interest rate and as a result debt rises at a faster rate.

## 2.5 Fiscal limits

### 2.5.1 Upper bound on debt

An increasing number of papers (among others Bi 2012; Bi et al. 2010, 2013; and Cochrane 2011) assume that the government faces a fiscal limit on debt, defined as the maximum level of debt that a government can repay. The fiscal limit on debt is motivated by appealing to Laffer curves. Since taxes are distortionary, there is a limit to the tax revenue that the government can raise - the top of the Laffer curve. Following Bi (2012), we assume that there is an upper bound, which we label  $\hat{d}$ , on the expected present value of the future primary surpluses that the government can raise

$$E_t \sum_{k=1}^{\infty} s_{t+k} \left( \prod_{i=1}^k \frac{1}{1 + r_{t+i}} \right) \leq \hat{d}.$$

Using the government's IBC, equation (4), this implies that there is a maximum level on the size of debt relative to GDP that the government is able to repay such that

$$d_t \leq \hat{d}. \quad (13)$$

The fiscal limit on debt has implications for solvency. Solvency requires debt relative to GDP does not breach the fiscal limit on debt.

### 2.5.2 Upper bound on the change in primary surplus

We also assume that the government faces a fiscal limit on the change in the primary surplus, defined as the maximum adjustment of primary surplus that the government can attain. The *second* fiscal limit is in part due to the limited political will to raise taxes fast enough. Public demonstrations and even riots against austerity programs are evidence of the difficulties in the political process of raising the tax revenue rapidly. Bi (2012) and Bi et al. (2010, 2013) recognize that a government might not have the political will to adjust taxes immediately. However, they model the limited political will as a reduction in the fiscal limit on debt. In contrast, we model the limited political will with a second fiscal limit on the change in the primary surplus, which we label  $\Delta s^{\max}$ , such that

$$\Delta s_t = \min \left\{ \beta_0 + (\beta_1 - 1) s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max \left( 0, d_{t-1} - d' \right)^2 + \beta_4 \tilde{y}_{t-1} + \varepsilon_t^s + \nu_t, \Delta s^{\max} + \varepsilon_t^s + \nu_t \right\}. \quad (14)$$

Additionally, Schaechter et al. (2012) find that some countries are imposing limits on the adjustment of tax hikes and government spending cuts to debt. Therefore, the second fiscal limit captures some of the policy ideas currently being implemented.

## 2.6 Criteria for fiscal solvency

Bohn (1998) was the first to explore theoretically the nonlinearities in the primary surplus-debt relationship and argued that  $\beta_2 + 2\beta_3 (d_{t-1} - d') > 0$  for  $d_{t-1} > d'$  was sufficient to yield a sustainable fiscal policy in the absence of fiscal limits. We now consider how a nonlinear fiscal rule coupled with fiscal limits changes Bohn's criteria. Davig (2005) and Daniel and Shiamptanis (2013) argue that the fiscal limit on debt requires boundedness of the debt relative to GDP. And when boundedness is added, a necessary condition for fiscal solvency

is that the dynamic model in the primary surplus and debt be globally stable. That is, the fiscal rule should satisfy the government's IBC, equation (4), and also rule out explosive behavior of debt relative to GDP. We derive the restrictions on the parameters of a nonlinear fiscal rule that yield a globally stable economy.

To determine the criteria for global stability in a nonlinear dynamic model, we compute the Jacobian matrix of the system<sup>10</sup> by subtracting the lagged value of the primary surplus from equation (11) and the lagged value of debt from equation (12) to yield

$$\begin{aligned}\Delta s_t &= s_t - s_{t-1} = \beta_0 + (\beta_1 - 1) s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max(0, d_{t-1} - d')^2 + \beta_4 \tilde{y}_{t-1} + \varepsilon_t^s + \nu_t, \quad (15) \\ \Delta d_t &= d_t - d_{t-1} = (r - \beta_2) d_{t-1} - \beta_0 - \beta_1 s_{t-1} - \beta_3 \max(0, d_{t-1} - d')^2 - \beta_4 \tilde{y}_{t-1} - \varepsilon_t^s - \nu_t - \gamma_t + E_{t-1} \gamma_t + \varepsilon_t^d.\end{aligned}\quad (16)$$

Let  $J$  denote the Jacobian matrix with shocks taking on their expected values of zero,  $\nu_t = \gamma_t = E_{t-1} \gamma_t = \varepsilon_t = 0$ , and is given by

$$J = \begin{pmatrix} \beta_1 - 1 & \beta_2 + 2\beta_3 \max(0, d_{t-1} - d') \\ -\beta_1 & (r - \beta_2) - 2\beta_3 \max(0, d_{t-1} - d') \end{pmatrix}.$$

The nonlinear system is globally stable if the trace  $J < 0 <$  determinant  $J$ , and also the off-diagonal entries of  $J$  are both non-zero and of opposite signs for all values of  $d_{t-1}$ . This requires

$$\frac{\partial s_t}{\partial d_{t-1}} = \beta_2 + 2\beta_3 \max(0, d_{t-1} - d') > r(1 - \beta_1) \quad \text{and} \quad (1 + r)\beta_1 < 1. \quad (17)$$

The consideration of nonexplosive behavior together with a nonlinear fiscal rule modifies Bohn's (1998) criterion. Our results suggest that for all values of debt the marginal response

<sup>10</sup>To determine global stability in linear dynamic models, the roots of the model are usually computed. To determine stability in nonlinear dynamic models, an alternative technique is employed, which requires the Jacobian matrix of the nonlinear model together with its trace and determinant. Please note that the Jacobian technique is equivalent to finding the roots of the dynamic model, without the need of explicitly computing those roots.

of the primary surplus to debt must be larger than the interest rate times one minus the persistence in the primary surplus. A positive, but small marginal response, which satisfies Bohn's criterion, fails to render a globally stable model and debt relative to GDP is expected to explode and violate any fiscal limit on debt.

When  $d_{t-1} < d'$ , our restrictions reduce to  $\beta_2 > r(1 - \beta_1)$ . The primary surplus responsiveness at low levels of debt is also important to eliminate explosive behavior. Additionally, equation (17) generalizes the original criteria on fiscal sustainability (Hamilton and Flavin 1986; Wilcox 1989; Trehan and Walsh 1991). When  $\beta_1 = \beta_3 = 0$ , as assumed in early work, our restrictions reduce to the original criteria, where the primary surplus must respond to debt by more than the interest rate.

## 2.7 Dynamics

If the criteria in equation (17) are satisfied, then explosive behavior is eliminated and the system is expected to reach its long-run equilibrium. It is useful to illustrate the dynamic behavior of the primary surplus and debt using phase diagrams, which reveal the direction of movement of the primary surplus and debt at each point. The phase diagram for equation (15) and (16) with  $\nu_t = \gamma_t = E_{t-1}\gamma_t = \varepsilon_t^d = \tilde{y}_{t-1} = 0$ , is presented in Figure 1. Debt is on the vertical axis and the primary surplus is on the horizontal axis. The  $\Delta s = 0$  and  $\Delta d = 0$  schedules are nonlinear for values of debt above the threshold debt level, and linear for values of debt below the threshold debt level,  $d'$ . The  $\Delta s = 0$  and  $\Delta d = 0$  schedules intersect at point G with  $s_t = rd_t = rd^*$ , where  $d^* = \frac{\beta_0}{r - r\beta_1 - \beta_2} \leq d' < \hat{d}$ . Point G represents the long-run equilibrium of equations (15) and (16) in which the primary surplus pays interest rate on debt, and debt equals its target level,  $d^*$ . This is a globally stable system, implying that

debt and the primary surplus are expected to reach their long-run equilibrium for any initial values.

## 2.8 Implications of a nonlinear fiscal policy rule

The dynamic behavior of debt and the primary surplus is illustrated in Figure 1, which shows the adjustment paths from point B under a linear and a nonlinear fiscal rule, labeled BHG and BDG, respectively. Although the two adjustment paths appear similar, there are two primary differences. First, given that the nonlinear  $\Delta d = 0$  schedule is below the linear  $\Delta d = 0$  schedule, in the area between the two  $\Delta d = 0$  schedules debt increases under a linear fiscal rule whereas debt decreases under a nonlinear rule. The debt is expected to attain a lower maximum value in its approach to the long-run equilibrium under a nonlinear fiscal rule. As a result, point D occurs at a lower level than point H. Second, the speed of adjustment is faster under a nonlinear fiscal rule. Given the increasing marginal response of the primary surplus to debt as debt rises under a nonlinear fiscal rule, the system moves along the path BDG much faster compared to the linear case (BHG). To summarize, if the responsiveness below the threshold level,  $\beta_2$ , is the same for both the linear and nonlinear fiscal rules, the stronger nonlinear response at high levels of debt acts as an additional stabilizing force. It reduces debt and risk to a greater extent as compared to a linear rule.

It is important to note that countries could exhibit different nonlinearities. Ghosh et al. (2013) use a nonlinear fiscal rule in which the responsiveness weakens as debt increases, a phenomenon that they coined as "fiscal fatigue." In our model, a negative coefficient on the nonlinear term,  $\beta_3 < 0$ , is sufficient for fiscal fatigue.<sup>11</sup> Ghosh et al. (2013) argue that

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<sup>11</sup>A negative  $\beta_3$  implies a decreasing marginal response to debt as debt increases above its threshold value.



a nonlinear fiscal rule that exhibits fiscal fatigue provides information about the maximum level of debt relative to GDP beyond which the debt explodes. There is one major difference between the Ghosh et al. (2013) type of fiscal limit on debt and the Bi (2012) and Bi et al. (2010, 2013) type of fiscal limit on debt, which is used in this paper. Ghosh et al. (2013) use a model in which the nonlinear fiscal rule is unstable and the dynamics eventually become explosive. A crucial assumption under their framework is that the fiscal limit exists only when  $\beta_3 < 0$ . This raises the question as to whether or not countries, which follow a nonlinear fiscal rule with  $\beta_3 > 0$ , face a fiscal limit on debt, and subsequently if a solvency crisis occurs. Bi (2012) and Bi et al. (2010, 2013) assume that the fiscal limit on debt is the sum of the present value of expected maximum primary surpluses, therefore a country with  $\beta_3 > 0$  still faces a fiscal limit and could experience a solvency crisis. In this paper, we focus on solvency crisis under a nonlinear fiscal rule which eliminates explosive behavior,  $\beta_3 > 0$ .

## 2.9 How will a solvency crisis occur?

Solvency is defined as the government's ability to repay its debt without violating its fiscal limit on debt. The criteria in equation (17) are necessary, but not sufficient, for solvency. A nonlinear fiscal rule that satisfies equation (17) eliminates explosive behavior of debt relative to GDP, but it does not assure that all paths approaching the long-run equilibrium respect the fiscal limit on debt. Consider the viability of a nonlinear fiscal rule, which satisfies the criteria in equation (17), using Figure 1. The Bi (2012) type of fiscal limit on debt is given by  $\hat{d}$  on the vertical axis. Assume that the initial adjustment path is AG. A negative shock moves the system in a northwest direction from its initial path. Consider a string of negative fiscal and productivity shocks which eventually move the system to point B along

the adjustment path BDG. Although path BDG eliminates explosive behavior, the dynamics imply that the value of debt along the path BDG is expected to pass through points where it exceeds the fiscal limit on debt. BDG is not an equilibrium path because it violates solvency requirements. Restoration of equilibrium requires a response. We consider policy switching and assume that agents know the response.<sup>12</sup>

## 2.10 Equilibrium with Policy Switching

**Definition 1** *Given values for the foreign interest rate and foreign price level, an inflation target, the fiscal limit on debt and on the change in primary surplus, and an initial nonlinear passive fiscal rule (equation 6) with plans for switching in the event that the government cannot carry out the fiscal rule, an equilibrium is a set of time series processes for the primary surplus, debt, and debt devaluation due to inflation,  $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$ , such that the government's flow and intertemporal budget constraints (equations 3 and 4) hold, expectations are rational, debt is not expected to exceed its fiscal limit on debt, and agents expect to receive the return on assets determined by interest rate parity (equation 1).*

Initially the fiscal authority follows a nonlinear passive fiscal rule, allowing the monetary authority to follow the active Taylor rule in equation (5). When the government is faced with an inability to borrow, the initial policy mix (passive regime) is not viable. To restore expectations of solvency and lending, the fiscal authority switches to an active fiscal policy (active regime) with a new target level for debt, and the monetary authority switches to a passive policy of pegging the interest rate at a value consistent with an inflation target of zero. The policy switching model considered here differs from the model of Daniel and Shiamptanis (2012) where authorities remain in the active regime once they switch. In this model, the government can switch back to the passive regime when the primary surplus is large enough to lower debt. Additionally, this policy switching model differs from the Markov

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<sup>12</sup>Davig et al. (2011) show that policy uncertainty affects inflation dynamics. Fernandez-Villaverde et al. (2012) show that fiscal volatility, defined as the greater-than-usual uncertainty about the future path of fiscal policy, reduces economic activity.

policy switching models of Davig and Leeper (2011) and Davig et al. (2010, 2011) where the policy switches are exogenous. In this model, switching is endogenous to the model. The timing of the policy switch to the active regime is determined by the country's inability to borrow from the markets under the passive regime. The timing of the switch back to the passive regime is determined by the country's ability to return to the markets under the passive regime.

Before analyzing the policy switch, it is necessary to present the path under the new policy mix (active regime). Under an active fiscal policy, the system travels along the upward-sloping saddlepath CKE, as shown in Figure 2. The saddlepath relationship between debt and the primary surplus, which is derived in Appendix A, can be expressed as

$$d_t^{sp} = \frac{\beta_1}{1 - \beta_1 + r} s_t + \frac{(1 - \beta_1)(1 + r)}{1 - \beta_1 + r} \hat{d}. \quad (18)$$

This is a saddlepath stable system, in which the government IBC is satisfied only for positions on the saddlepath. Negative shocks move the system away from the saddlepath and debt devaluations,  $\gamma_t$ , move the system back to the saddlepath. The post-crisis equilibrium is characterized by the Fiscal Theory of the Price Level. This policy ensures solvency by having the real outstanding value of debt to adjust through price level jumps. However, this regime creates price instability.

In this paper, the government can switch back to the passive regime when the primary surplus is adequate to lower debt. This occurs along the KE portion of the saddlepath. The saddlepath, given by equation (18), and the nonlinear  $\Delta d = 0$  schedule, given by equation (16), intersect at point K, and the value of the primary surplus at point K is given by

$$s^K = \frac{1 - \beta_1 + r}{\beta_1} d^K - \frac{(1 - \beta_1)(1 + r)}{\beta_1} \hat{d},$$

where  $d^K = \frac{-\beta_2 - 1 + \beta_1 + 2\beta_3 d' + [(\beta_2 + 1 - \beta_1 - 2\beta_3 d')^2 - 4\beta_3(\beta_0 + \beta_3 d'^2 - (1 - \beta_1)(1 + r)\hat{d})]^{1/2}}{2\beta_3}$  is the value of debt at point K. For values of the primary surplus above  $s^K$ , the slope of any passive adjustment path becomes negative. In this region, under a passive regime the primary surplus is large enough such that the dynamics are lowering government debt and further raising the primary surplus. Therefore, once the economy along CKE passes point K, the government switches back to its initial policy mix with no price change.

### 2.10.1 Fiscal Space

The maximum value of debt consistent with solvency under the expectation of resolving a crisis with policy switching is given by

$$d_t^{\max} = \begin{cases} d_t^{sp} & \text{for } s_{t-1} \leq s^k \\ \hat{d} & \text{for } s_{t-1} \geq s^k \end{cases}. \quad (19)$$

For values of the primary surplus less than  $s^k$ , any path above the upward-sloping saddlepath violates the fiscal limit on debt and is inconsistent with solvency. For values of the primary surplus greater than  $s^k$ , debt at its fiscal limit is consistent with solvency because the adjustment dynamics under the passive regime suggest that future debt falls.

Equations (11), (12), (14), (18), and (19) can be used to express the fiscal space,  $\Omega_t$ , between  $d_t^{\max}$  and the current value of debt,  $d_t$ , as

$$\Omega_t = d_t^{\max} - d_t = \begin{cases} \frac{(1+r)}{1-\beta_1+r} (x_{t-1} + \varepsilon_t^s + \nu_t) - \varepsilon_t^d + \gamma_t - E_{t-1}\gamma_t & \text{for } s_{t-1} \leq s^k \\ x_{t-1} + \varepsilon_t^s + \nu_t - \varepsilon_t^d + \gamma_t - E_{t-1}\gamma_t & \text{for } s_{t-1} \geq s^k \end{cases}$$

where  $x_{t-1}$  is the state variable determining the fiscal space and is given by

$$x_{t-1} = \begin{cases} \left[ \min \left\{ \beta_0 + \beta_1 s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max (d_{t-1} - d')^2 + \beta_4 \tilde{y}_{t-1}, \Delta s^{\max} \right\} \right. \\ \quad \left. - r d_{t-1} + (1 - \beta_1) (\hat{d} - d_{t-1}) \right] & \text{for } s_{t-1} \leq s^k \\ \left[ \min \left\{ \beta_0 + \beta_1 s_{t-1} + \beta_2 d_{t-1} + \beta_3 \max (d_{t-1} - d')^2 + \beta_4 \tilde{y}_{t-1}, \Delta s^{\max} \right\} \right. \\ \quad \left. - r d_{t-1} + \hat{d} - d_{t-1} \right] & \text{for } s_{t-1} \geq s^k \end{cases} . \quad (20)$$

We can write a general expression for the fiscal space as

$$\Omega_t = c(x_{t-1} + u_t) + \gamma_t - E_{t-1}\gamma_t \quad (21)$$

where  $c$  is a constant and  $u_t$  is the total impact of the fiscal and productivity shocks on the fiscal space

$$c = \begin{cases} \frac{1+r}{1-\beta_1+r} & \text{for } s_{t-1} \leq s^k \\ 1 & \text{for } s_{t-1} \geq s^k \end{cases} \quad \text{and} \quad u_t = \begin{cases} \varepsilon_t^s + \nu_t - \frac{1}{c}\varepsilon_t^d & \text{for } s_{t-1} \leq s^k \\ \varepsilon_t^s + \nu_t - \varepsilon_t^d & \text{for } s_{t-1} \geq s^k \end{cases} .$$

We define a shadow value of debt devaluation via inflation,  $\tilde{\gamma}_t$ , which represents the reduction in the value of debt needed for the economy to reach equation (19). Setting  $\Omega_t$  to zero in equation (21) yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - c(x_{t-1} + u_t). \quad (22)$$

Substituting into equation (21) yields an expression for  $\Omega_t$  as

$$\Omega_t = \gamma_t - \tilde{\gamma}_t,$$

implying that when the shadow value of debt devaluation via inflation is positive,  $\tilde{\gamma}_t > 0$ , the fiscal space is negative,  $\Omega_t < 0$ .

Agents refuse to lend, creating a crisis if  $\tilde{\gamma}_t > 0$ . Policy switching with debt devaluation restores equilibrium. We relegate all the proofs in Appendix A and focus here on intuition.

Negative shocks,  $u_t < 0$ , and expectations of debt devaluation through inflation,  $E_{t-1}\gamma_t > 0$ , could push the economy over the maximum level of debt, equation (19). Point  $B$  is very close to the saddlepath and the markets begin to anticipate policy switch together with inflation. This anticipation raises the domestic interest rate, from the interest rate parity equation (1), to accommodate for the expectations of debt devaluation through inflation. Therefore, debt is expected to increase more quickly than implied by the initial policy locus  $BD$ , as indicated by the arrow from point  $B$  in Figure 3, and could breach its maximum level. Switching to the active regime with debt devaluation,  $\gamma_t = \tilde{\gamma}_t$ , lowers debt and restores lending.

Additionally, a crisis occurs if  $x_t < 0$ . In the current period, the economy could reach the saddlepath  $CK$  without going over,  $\Omega_t = 0$ , however in the next period the dynamics would push the economy over the saddlepath  $CK$ ,  $x_t < 0$ . This occurs because for values of the primary surplus below  $s^k$ , the passive adjustment path crosses the active saddlepath. Switching to the active regime without debt devaluation in the current period restores equilibrium.

After the policy switch, the system travels along the saddlepath. Fiscal and productivity shocks move the system away from the saddlepath and debt devaluations and revaluations via price level changes,  $\gamma_t$ , move the system back to the saddlepath. The passive adjustment path  $JKG$ , which peaks at point  $K$ , represents the highest path that does not require policy switching. Once the economy along  $CKE$  passes point  $K$ , the authorities switch back to the passive regime without any debt devaluations. The fiscal and monetary authorities resume their original roles. Expectations of devaluations fade away and the system is expected to travel along the passive adjustment path towards its long-run equilibrium, point  $G$ .

### 3 Model Applied: The case of Canada

In this section, we apply the model to the Canada, a country which has shown that its primary surplus responsiveness changes at higher levels of debt. The behavior of the primary surplus and debt relative to GDP between 1970 and 2012 are presented in Figure 3. Although most of the recent focus has been on European countries, soaring debt levels are a major concern for most developed countries. Canada's current debt level breached the critical level of 90% of GDP, which back in 1992 resulted in losing its AAA debt rating. Additionally, the Canadian fiscal policy is not immune to future stochastic shocks. Canada's main trading partners, US and Europe, are facing many challenges.

The purpose of this section is to first estimate the parameters of the fiscal rule for Canada using annual data for the period of 1970-2012 from the OECD database. The second purpose is to quantify the probability of a solvency crisis. We use the estimates of the fiscal rule, together with the 2012 values for the Canadian primary surplus and debt relative to GDP, and we simulate the nonlinear dynamic model.<sup>13</sup>

#### 3.1 Estimation

We follow Hansen's (2000) procedure who recommends estimation of  $d'$  by minimizing the sum of squared errors,  $SSE$ , of equation (6),  $d' = \arg \min_{d'} SSE(d')$ . Once  $d'$  is obtained, the fiscal policy parameters are estimated using least squares and White robust standard errors to address potential concerns about heteroskedasticity. We find that the Canadian threshold

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<sup>13</sup>To study countries like Canada in which fiscal fatigue is not observed, Ghosh et al. (2013) use a panel of countries. This allows them to use information from countries that exhibit fiscal fatigue to infer fiscal fatigue for countries in which fiscal fatigue is not observed. Here we take a different approach. We focus on a single country that has been following a globally stable fiscal reaction function and combine it with Bi's (2012) fiscal limit to study the possibility of future insolvency.

value of debt is at 89.52% of GDP and the fiscal policy parameters are presented in Table 1 under Regression 1.<sup>14</sup>

**Table 1: Estimates of the fiscal rule**

parameters	(1)	(2)
$\beta_1$	0.761*** (0.057)	0.740*** (0.057)
$\beta_2$	0.029** (0.015)	0.044*** (0.011)
$\beta_3$	0.006*** (0.002)	
$\beta_4$	0.400*** (0.091)	0.367*** (0.091)
$\bar{R}^2$	0.869	0.858
$\sigma$	1.225	1.275
<i>AIC</i>	3.334	3.392
<i>BIC</i>	3.499	3.516

Note: The \*\* and \*\*\* indicate statistical significance at the 95 and 99 percent level, respectively.

The estimates for  $\beta_2$  and  $\beta_3$  in Regression 1 reveal that the Canadian fiscal rule is systematically responding to debt. The coefficient on linear  $d_{t-1}$  is significantly positive and similar in magnitude to those obtained by Bohn (1998, 2008) and Mendoza and Ostry (2008).<sup>15</sup> The coefficient on the spline term is also positive and enters significantly at the 99 percent confidence level. The coefficient on cyclical fluctuations in GDP enters significantly, with the expected sign and is consistent with the tax smoothing literature. Our results are robust to alternative estimation techniques and to the inclusion of various control variables presented in Appendix B.

Additionally, the results satisfy the criteria in equation (17). We find that the marginal

<sup>14</sup>All the variables are from the OECD database (OECD Economic Outlook No. 94). For  $s_t$  we use the general government primary balances relative to GDP, for  $d_t$  we use the general government gross financial liabilities relative to GDP, and for  $\hat{y}_t$  we use the economy's output gap.

<sup>15</sup>Bohn's (1998) estimates for  $\beta_2$  using US data over 1916-1995 range from 0.028 to 0.054. Bohn's (2008) estimates for  $\beta_2$  using US data over 1792-2003 range from 0.028 to 0.147. Mendoza and Ostry (2008) estimates for  $\beta_2$  using 22 industrial countries over 1970-2005 range from 0.020-0.038 and using 34 emerging economies over 1990-2005 range from 0.035 to 0.106.



response of the primary surplus to debt is always larger than  $r(1 - \beta_1)$ .<sup>16</sup> Moreover, the positive estimates for  $\beta_2$  and  $\beta_3$  in Regression 1 suggest that the marginal response of the primary surplus to debt is increasing in the debt level, for instance rising from 0.029 when debt is below the threshold level to 0.091 when debt is at 95% of GDP, and rising further to 0.148 when debt is at 100% of GDP.<sup>17</sup>

The importance of the spline term is best illustrated by comparing Regression 1 to Regression 2 that excludes the nonlinear variable. Given that the spline term enters significantly in Regression 1 and also increases  $\bar{R}^2$ , while decreasing  $\sigma$  and minimizing the Akaike information criterion,  $AIC$ , and Schwarz Bayesian information criterion,  $BIC$ , yields evidence that the fiscal rule characterizing the Canadian policy is indeed nonlinear.

Linear rules have been criticized for being backward-looking and not capturing changes in fiscal policy. Under a linear rule, the future marginal response of the primary surplus is assumed to be the same as the historical marginal response. Davig et al. (2007) argued that there are regime shifts over different periods in the US. Similarly, Lloyd-Ellis and Zhu (2001) argued that there was a change in the fiscal policy of the Canadian government in the mid-1990s when debt was elevated. These papers assume two linear policy rules with different parameters and allow stochastic switching over time between them. In contrast, we use a nonlinear fiscal rule, which naturally yields a stochastic marginal response of the primary surplus to debt for values of debt bigger than  $d'$ . Therefore, changes in the fiscal policy at high levels of debt can be attributed to the stochastic responsiveness of the primary surplus to debt under a nonlinear rule, rather than the stochastic switching between two linear rules.

<sup>16</sup>The results are robust to the choice of  $r$ . We obtain identical conclusions when the growth-adjusted interest,  $r$ , is anywhere between 0% to 10%.

<sup>17</sup>When  $d_{t-1} = 95\%$  of GDP,  $\frac{\partial s_t}{\partial d_{t-1}} = 0.029 + 2(0.006)(95 - 89.52) \approx 0.091$ .

In summary, our results reveal that the nonlinear fiscal rule characterizing Canadian policy satisfies our criteria. However, in a stochastic world, a fiscal rule which eliminates explosive behavior is not sufficient to assure the absence of a solvency crisis because any country could experience negative shocks. We turn to this below.

## 3.2 Simulations

In this section, we quantify the probability of a solvency crisis over the next decade. Given estimates for the fiscal rule, the distribution of shocks, and policy-switching as the method of crisis resolution, the dynamic nonlinear model can be solved numerically and simulated to estimate the probability of a solvency crisis, and equivalently the probability of price instability. The simulation algorithm is given in Table 3.

For the simulations, we use the parameter estimates from Regression 1 in Table 1. Under the assumption that fiscal and productivity shocks are correlated and both have a normal distribution with mean zero, the standard deviation of fiscal shocks is set at 1.23% of GDP, which is the estimate of the standard error of Regression 1, and the standard deviation of productivity shocks is set at 2.12% of GDP, which is the estimate of the standard error of equation (7). The correlation is set at 0.47, which is the estimate of the correlation coefficient between the residuals of Regression 1 and equation (7). Further, we let the lower and upper bound on the fiscal and productivity shocks correspond to two standard deviations.<sup>18</sup> Additionally, the real interest rate,  $i$ , is set at 4%,<sup>19</sup> which is the the average value of the long-run real rate on government bonds over the sample period, the real long-run output growth rate,  $\rho$ , is set at 2.5%, which is the estimate of  $\rho$  from equation (7), the target

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<sup>18</sup>We set bounds on the distributions of the shocks to avoid skewing the results with draws close to  $\pm\infty$ .

<sup>19</sup>We exogenously set the risk-free real interest rate, but the expectations of inflation which in turn raise the domestic risky interest rate are endogenous to the model.

debt value,  $d^*$ , is set at 75% of GDP, which is the average value of debt relative to GDP over the sample period, the fiscal limit on debt,  $\hat{d}$ , is set at 155% of GDP to match Bi's (2012) mean estimate for Canada,<sup>20</sup> and the second fiscal limit,  $\Delta s^{\max}$ , is set at 6% of GDP, which is the largest change in the primary surplus relative to GDP among the G7 countries over the sample period.

To determine the safety of Canada under the current fiscal state, we simulate the model using the 2012 values of debt to GDP and the primary surplus to GDP levels.<sup>21</sup> We find that under the baseline parameters values, Canadian fiscal policy is perfectly safe with no solvency crises over ten years. This implies that the 2012 level of Canadian debt to GDP ratio is adequately low and its nonlinear fiscal rule is strong enough that there should be no concerns about a solvency crisis.

Given that the OECD is projecting that in the next couple of years the Canadian fiscal position will deteriorate, we consider how the probability of a solvency crisis changes as the initial value of debt,  $d_{t-1}$ , increases from its 2012 value and the initial value of primary surplus,  $s_{t-1}$ , decreases from its 2012 value. Figure 4 illustrates that both the level of the primary surplus and debt are significant factors to solvency risk. Lower primary surplus to GDP ratios and higher debt to GDP ratios could substantial increase the probability of solvency crisis. To examine the relationship between the primary surplus and the probability of solvency crisis, we dissect Figure 4 and look at the cross section when debt is at 150% of GDP. We find that the crisis probability is 100% if  $s_{t-1}$  is below -8.5% of GDP, and it drops to 2.4% once  $s_{t-1}$  increases to 0% of GDP as shown in Figure 5. A government could substantially reduce the probability of a solvency crisis when the primary deficit is reduced.

<sup>20</sup>For Canada, Ghosh et al. (2013) estimate for  $\hat{d}$  is 152.5% of GDP when the historical interest rate is used.

<sup>21</sup>Source:  $d_{t-1} = 96.11\%$  and  $s_{t-1} = -2.83\%$ . OECD Economic Outlook No. 94

To examine the relationship between the debt level and the probability of solvency crisis, we slice Figure 4 and study at the cross section when the primary surplus is at -5% of GDP. We find that the crisis probability becomes positive once  $d_{t-1}$  exceeds 128% of GDP, and unity once  $d_{t-1}$  exceeds 153.5% of GDP, as shown in Figure 6. To illustrate how sensitive the probability of a crisis is to the inclusion of the nonlinear term, we repeat the simulations under a linear fiscal rule. We set  $\beta_3 = 0$  and find that at the 2012 debt level the probability of a crisis is 21.6% and as shown in Figure 6 the crisis probability becomes unity once  $d_{t-1}$  exceeds 138% of GDP. Our results suggest that the nonlinear term is reducing the probability of a solvency crisis substantially. The opposite also holds. If the fiscal rule was linear with  $\beta_2 = 0.029$  and at the same time debt started creeping upwards, the Canadian government could face solvency risks even when the economy is well below the fiscal limit.

Perhaps if the Canadian fiscal authority did not respond nonlinearly, it would be more aggressive for all values of debt. The model implies that a linear fiscal rule with a larger primary surplus responsiveness,  $\beta_2$ , will reduce the probability of a solvency crisis. We repeat the simulations under a linear rule with  $\beta_2 = 0.044$ , which is the estimate of  $\beta_2$  from Regression 2 in Table 1, and find that at the 2012 debt level the probability of a crisis is 12% and the crisis probability becomes unity once debt exceeds 142.5% of GDP. We perform the same analysis using larger values for  $\beta_2$  and find that a linear rule with  $\beta_2 = 0.096$  yields the same results as a nonlinear fiscal rule with  $\beta_2 = 0.029$  and  $\beta_3 = 0.006$ , as shown in Figure 6.

Our results imply that under a linear fiscal rule, the value of the policy parameter  $\beta_2$  is extremely important to crisis probabilities. Next we consider how important  $\beta_2$  is under a nonlinear fiscal rule. We investigate how crisis probability changes as  $\beta_2$  declines. We use debt at 145% of GDP and the primary surplus at -5% of GDP to illustrate since the

probability of a solvency crisis is nonzero (3.7%). Figure 7 plots the probability of a solvency crisis of a nonlinear fiscal rule as a function of  $\beta_2$ . The results show that under the baseline parameter of  $\beta_3 = 0.006$ , the crisis probabilities is virtually unchanged as  $\beta_2$  declines. We find that when  $\beta_2$  is reduced from its baseline value of 0.029 to the smallest value of  $r(1 - \beta_1) = 0.004$  that guarantees global stability, the probability of a crisis upticks from 3.7% to 4.0%. This suggest that a nonlinear fiscal rule with  $\beta_2 = 0.004$  and  $\beta_3 = 0.006$  is almost as risky as a linear rule with  $\beta_2 = 0.096$ . Our results imply that being aggressive at low levels of debt is not necessary as it does not fundamentally alter the crisis probabilities. Even when the coefficient on the nonlinear term,  $\beta_3$ , is reduced by two-standard deviations to 0.002, smaller values of  $\beta_2$  have miniscule impact on the crisis probabilities. When  $\beta_2$  declines from 0.029 to 0.004, the probability of a crisis edges up from 40.6% to 40.9% (less than half a percentage point). Our results imply that a country that follows a nonlinear fiscal rule could substantially reduce its responsiveness below the threshold debt level without essentially raising the possibility of future insolvency.

Next we consider several sensitivity analysis scenarios. These include changing parameter values one at a time in the risky direction.<sup>22</sup> Experiments include: (A) raising  $\beta_1$ , which implies higher persistence most likely stemming from the rigidity of adjusting taxes and expenditure, (B) increasing  $i$ , which implies higher world real interest rates, (C) decreasing  $\rho$ , which implies a deceleration in real potential GDP growth, (D) raising  $d'$ , which implies that the nonlinear term takes effect at higher levels of debt, and (E) lowering  $\Delta s^{\max}$ , which implies that the fiscal authority is more constraint.<sup>23</sup> While the crisis probabilities are higher

<sup>22</sup>The sensitivity analysis scenarios also capture the estimation uncertainty as the estimated parameters could differ from the true parameters.

<sup>23</sup> $\beta_1$  is increased by one standard deviation,  $i$  is raised from 4% to 5.5%,  $\rho$  is reduced from 2.5% to 2%,  $d'$  is raised from 89.52% to 95%, and  $\Delta s^{\max}$  is reduced from 6% to 2.5%, which is the largest change in the

under Experiment A - E as shown in Figure 7, the implications are identical to the ones under baseline parameters. A country could design a nonlinear fiscal policy rule, which requires a very aggressive response only at high levels of debt, to yield the same risk as a linear fiscal rule, which requires an aggressive responsiveness for every debt level. The strong responsiveness of the nonlinear fiscal rule is vital to reduce the probability of a solvency crisis, however this strength need not be exhibited at low levels of debt. The responsiveness at low levels of debt is not very important to risk as long as the government has a higher responsiveness at high levels of debt. A nonlinear fiscal rule allows a country to reduce its responsiveness below the threshold debt level without essentially raising the probability of a solvency crisis.

## 4 Conclusion

The recent financial crisis highlighted the need to design better fiscal rules for the future. Many countries are implementing nonlinear fiscal rules, which require the responsiveness of the primary surplus to strengthen at higher levels of debt. In this paper, we study the implications of a nonlinear fiscal rule coupled with fiscal limits on solvency crisis. First, we derive conditions for a nonexplosive equilibrium and find that for all values of debt the marginal response of the primary surplus to debt should be larger than the one proposed by Bohn (1998). However, a government which follows a nonexplosive nonlinear fiscal rule could still experience a solvency crisis because of negative shocks. Second, we derive the dynamics leading to a solvency crisis under a nonlinear fiscal policy rule. We find that the stronger nonlinear response of primary surplus to debt acts as an extra stabilizing force. It

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primary surplus in Canada over the sample period.

lowers the expected maximum level of debt on a path towards its long-run target and also shortens the adjustment time. Third, we apply the model to Canada, a country which has shown that it is willing to become aggressive in its primary surplus responsiveness to debt. We estimate the parameters of the nonlinear policy rule over the period of 1970-2012, and then use them to quantify the probability of a solvency crisis. We find a nonlinear fiscal rule allows a country to reduce the probability of a solvency crisis without large responsiveness at early levels of debt.

## 5 Appendix A: Equilibrium with policy switching

Here we extend the dynamic policy-switching model of Daniel and Shiamptanis (2012) to have a nonlinear fiscal rule, stochastic output growth rate and to allow for switching back to the initial policy mix. Before analyzing the timing of the policy switch, it is necessary to present the post-crisis policy mix.

### 5.1 The Post-Crisis Policy Mix: Active Fiscal and Passive Monetary Policy

Under an active fiscal policy, the primary surplus does not respond to debt. We model this by setting  $\beta_2 = \beta_3 = 0$  in equations (6). We also allow the government to revise its debt target to a higher level. If the revised target value of debt is the fiscal limit,  $\hat{d}$ ,<sup>24</sup> the dynamic equations of the model become

$$\Delta s_t = s_t - s_{t-1} = (1 - \beta_1) \left( r\hat{d} - s_{t-1} \right) + \beta_4 \tilde{y}_t + \nu_t,$$

$$\Delta d_t = d_t - d_{t-1} = (rd_{t-1} - s_{t-1}) - (1 - \beta_1) \left( r\hat{d} - s_{t-1} \right) - \beta_4 \tilde{y}_t - \nu_t - \gamma_t + E_{t-1} \gamma_t + \varepsilon_t^d.$$

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<sup>24</sup>We show below that this assumption gives the government the maximum probability of being able to sustain its initial policy mix.

The eigenvalues of the dynamic system under an active fiscal policy are  $1 + r$  and  $\beta_1$ . This is a saddlepath-stable system, in which the government's IBC is not satisfied for positions off the saddlepath, and therefore is not satisfied for any initial value of debt, and hence for any inflation rate.

The time paths for primary surplus and debt relative to output under the active-fiscal and passive-monetary regime are given by

$$s_t = r\hat{d} + \beta_1^t \left[ s_0 - r\hat{d} + \sum_{k=1}^t \beta_1^{-k} (\nu_k + \beta_4 \tilde{y}_k) \right],$$

$$d_t = \hat{d} + \beta_1^t \left( \frac{\beta_1}{1 - \beta_1 + r} \right) \left[ s_0 - r\hat{d} + \sum_{k=1}^t \beta_1^{-k} (\nu_k + \beta_4 \tilde{y}_k) \right].$$

These equations can be used to express the saddlepath relationship between debt and the primary surplus as

$$d_t^{sp} = \frac{\beta_1}{1 - \beta_1 + r} s_t + \frac{(1 - \beta_1)(1 + r)}{1 - \beta_1 + r} \hat{d}.$$

The dynamics feature an upward-sloping linear saddlepath leading to  $\hat{d}$ , labeled CE in Figure 2.

## 5.2 Solvency crisis resolved with switching

### 5.2.1 Expected Inflation

To solve for expected debt devaluation due to inflation ( $E_{t-1}\gamma_t$ ), or for short, expected inflation, we assume that agents believe that a solvency crisis will occur if  $\tilde{\gamma}_t \geq 0$ . We show that this assumption is consistent with a rational expectations equilibrium below in Proposition 2. The government responds to the crisis with policy-switching, and when  $\tilde{\gamma}_t > 0$ , inflation reduces debt to equation (19). This implies that the equilibrium value for inflation



in period  $t$  is given by

$$\gamma_t = \max \{ \tilde{\gamma}_t, 0 \} = \max \{ E_{t-1} \gamma_t - c(x_{t-1} + u_t), 0 \}, \quad (23)$$

where we have used equation (22) to substitute for  $\tilde{\gamma}_t$ . To solve for the magnitude of inflation,  $\gamma_t$ , we must first solve for expectations of inflation,  $E_{t-1} \gamma_t$ .

Define  $u_t^*$  as a critical value for  $u_t$  such that  $\gamma_t > 0$  for  $u_t < u_t^*$ , and  $\gamma_t = 0$  for  $u_t \geq u_t^*$ . Letting  $f(u_t)$  be a bounded, symmetric, mean-zero distribution for  $u_t$ , with bounds given by  $\pm \bar{u}$ , the probability of a crisis can be expressed as

$$F(u_t^*) = \int_{-\bar{u}}^{u_t^*} u_t f(u_t)$$

and the expectation for (23) can be expressed as

$$E_{t-1} \gamma_t = \int_{-\bar{u}}^{u_t^*} \gamma_t f(u_t) = \int_{-\bar{u}}^{u_t^*} (E_{t-1} \gamma_t - c(x_{t-1} + u_t)) f(u_t).$$

Collecting terms on the expectation yields

$$[1 - F(u_t^*)] E_{t-1} \gamma_t = -c \left[ x_{t-1} F(u_t^*) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) \right]. \quad (24)$$

Substituting into equation (23), yields an implicit expression for  $\gamma_t$

$$[1 - F(u_t^*)] \gamma_t = -c \left[ x_{t-1} + u_t (1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) \right] \quad (25)$$

**Proposition 1** *Under the initial policy mix with plans for switching, an equilibrium solution for expected inflation ( $E_{t-1} \gamma_t$ ) exists if and only if the state variable determining the distance to the saddlepath at time  $t$  is greater than or equal to zero ( $x_{t-1} \geq 0$ ).*

**Proof.** We prove that there is no value for  $E_{t-1} \gamma_t$  when  $x_{t-1} < 0$ . To determine the probability of a crisis,  $F(u_t^*)$ , and expectations of debt devaluation,  $E_{t-1} \gamma_t$ , first solve for the largest value of total shock which will create a crisis,  $u_t^*$ .

A solution for  $u_t^*$  exists if and only if there exists a value for  $u_t^*$ , satisfying  $-\bar{u} \leq u_t^* \leq \bar{u}$ , which sets  $x_{t-1} + u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) = 0$  such that  $\gamma_t = 0$ . We prove that the term  $u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) \leq 0$  for all feasible values for  $u_t^*$ . Let  $u_t^*$  take on its smallest possible value of  $-\bar{u}$ . Then  $u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) = -\bar{u} < 0$ . The derivative of the term with respect to  $u_t^*$  is given by  $1 - F(u_t^*)$ . For  $u_t^* < \bar{u}$ , the derivative is positive. Therefore, as  $u_t^*$  rises,  $u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t)$  rises monotonically. Once  $u_t^*$  takes on its largest possible value, given by  $\bar{u}$ ,  $1 - F(\bar{u}) = 0$ , and  $u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t)$  takes on its maximum value of zero. Therefore,  $u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) \leq 0$  for all feasible values of  $u_t^*$ . A necessary and sufficient condition for  $x_{t-1} + u_t^*(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t) = 0$  is that  $x_{t-1} \geq 0$ . Therefore, when  $x_{t-1} \geq 0$ , a solution for  $u_t^*$  exists and the expectations of devaluation are given by the solution of equation (24). The dynamic system must bound the system away from positions for which  $x_{t-1} < 0$ . ■

**Corollary 1** *If the policymakers want to design a post-crisis policy to allow the initial policy to continue as long as possible, they will revise target debt to its highest possible value, that is, at the fiscal limit,  $\hat{d}$ .*

**Proof.** This is because  $x_{t-1}$  from equation (20) is increasing in  $\hat{d}$ . ■

**Corollary 2** *Nonlinear fiscal rules with  $\beta_3 > 0$  decrease the probability of a solvency crisis.*

**Proof.** This is because  $x_{t-1}$  from equation (20) is increasing in the nonlinear term of the fiscal rule,  $\beta_3 \max(0, d_{t-1} - \hat{d})^2$ , when  $\beta_3 > 0$ . ■

### 5.2.2 When Will Creditors Refuse to Lend?

**Proposition 2** *Creditors refuse to lend, creating a solvency crisis, if  $x_t < 0$  or  $\tilde{\gamma}_t > 0$ . Policy-switching restores equilibrium and allows government borrowing.*

**Proof.** From Proposition 1, when the state variable determining the next period's fiscal space is negative,  $x_t < 0$ , there is no equilibrium solution for expected inflation that will

provide the market rate of return to creditors in the absence of policy switching. Therefore, policy switching restores equilibrium.

Now consider the case where  $\tilde{\gamma}_t > 0$ . Substituting equation (22) into (21) yields  $\Omega_t = \gamma_t - \tilde{\gamma}_t$ . When the shadow value for inflation is positive,  $\tilde{\gamma}_t > 0$ , but there is no policy switch with inflation,  $\gamma_t = 0$ , the fiscal space ( $\Omega_t$ ) is negative, an impossibility. In equilibrium debt cannot be above the saddlepath CE. Policy switching together with inflation,  $\gamma_t = \tilde{\gamma}_t$ , restores equilibrium. ■

## 6 Appendix B: Sensitivity analysis of the nonlinear fiscal rule

Our results are robust to alternative estimation methods. We use an instrumental variables (IV) procedure to correct for the potential endogeneity of output gap. Following Gali and Perotti (2003), we instrument  $\tilde{y}_t$  using its lagged value,  $\tilde{y}_{t-1}$ , and the current value for the U.S. output gap,  $\tilde{y}_t^{us}$ . Next, we use the fully-modified OLS (FMOLS) procedure of Phillips and Hansen (1990) to account for the possibility of nonstationary variables.<sup>25</sup> It is a semiparametric approach that adjusts for the effects of endogenous regressors and short-run dynamics of the errors. We use the Bartlett kernel and the Newey-West bandwidth selection procedure.<sup>26</sup> Table 2 shows that the estimates are similar to the estimates from Table 1.

The subsequent regressions use least squares estimation as in Table 1, and they illustrate the robustness of our results to the inclusion of additional variables. We begin by adding a cubic term. In contrast to Ghosh et al. (2013), its coefficient is positive. We then use a linear spline of the form  $\max(0, d_{t-1} - d')$ , as in Mendoza and Ostry (2008), rather than

<sup>25</sup>The ADF, PP, KPSS and Ng-Perron unit root tests do not provide conclusive results about the existence of nonstationarity.

<sup>26</sup>The FMOLS results are robust to the choice of kernel and bandwidth.

a square spline. Its coefficient is positive and statistically significant. However, a linear spline only allows for a one-time change in the marginal response, and once debt crosses its threshold level, the marginal response of the primary surplus to debt remains constant at  $\frac{\partial s_t}{\partial d_{t-1}} = \beta_2 + \beta_3$ , whereas a squared spline term yields a time-varying marginal response for  $d_{t-1} > d'$ . Finally, following Ghosh et al. (2013), we include a richer set of control variables as determinants of the primary surplus such as temporary changes in government outlays, inflation, trade openness, and oil prices.<sup>27</sup> The results illustrate that trade openness affects the primary surplus positively, whereas temporary government outlays and oil prices affect it negatively. The coefficient on inflation is not statistically different from zero.

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<sup>27</sup>All the variables are from the OECD database (OECD Economic Outlook No. 94). For temporary government spending, we use the cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter, for inflation we use the average inflation in the previous three years, for trade openness we use the sum of exports and imports relative to GDP, and for oil prices we use the trend component of the log oil prices, as in Ghosh et al. (2013).

**Table 2: Alternative estimations**

Variables	IV	FMOLS	LS		
			Cubic	Linear Spline	Control Var
$s_{t-1}$	0.762*** (0.063)	0.659*** (0.058)	0.702*** (0.049)	0.766*** (0.060)	0.550*** (0.049)
$d_{t-1}$	0.029** (0.014)	0.043** (0.017)	-0.027* (0.016)	0.028* (0.016)	0.022 (0.021)
$\max(0, d_{t-1} - d')$	0.006*** (0.002)	0.008** (0.003)	0.012*** (0.001)		0.003* (0.004)
$\tilde{y}_t$	0.394*** (0.122)	0.451*** (0.081)	0.404*** (0.087)	0.394*** (0.091)	0.375*** (0.054)
$(d_{t-1} - d')$			2.9E-05*** (9.7E-06)		
$\max(0, d_{t-1} - d')$				0.081** (0.038)	
government outlays					-0.307*** (0.067)
inflation					-0.102 (0.122)
trade					0.037*** (0.011)
oil					-0.806*** (0.154)
$\bar{R}^2$	0.869	0.921	0.889	0.866	0.953
$\sigma$	1.225	0.962	1.132	1.239	0.738

Note: The \*\* and \*\*\* indicate statistical significance at the 95 and 99 percent level, respectively.

**Table 3**


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**Simulation Algorithm: Probability of a solvency crisis over the next ten years**

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1. Compute the state variable determining the fiscal space,  $x_{t-1}$ , from equation (20) using initial values of debt/GDP,  $d_{t-1}$ , and the primary surplus/GDP,  $s_{t-1}$ .
2. Compute the expectations for inflation,  $E_{t-1}\gamma_t$ , from equation (24).
3. Draw a fiscal and a productivity shock
4. Calculate the value for capital loss due to inflation,  $\gamma_t$ , from equation (25).
5. If  $\gamma_t = 0$ , then next period's debt and primary surplus are updated using equations (3) and (6), which are then used to update  $x_t$ .
6. If  $\gamma_t > 0$  or  $x_t < 0$ , then there is a solvency crisis and the simulation ends.
7. If not, repeat steps 2-6 for ten years.
8. We repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

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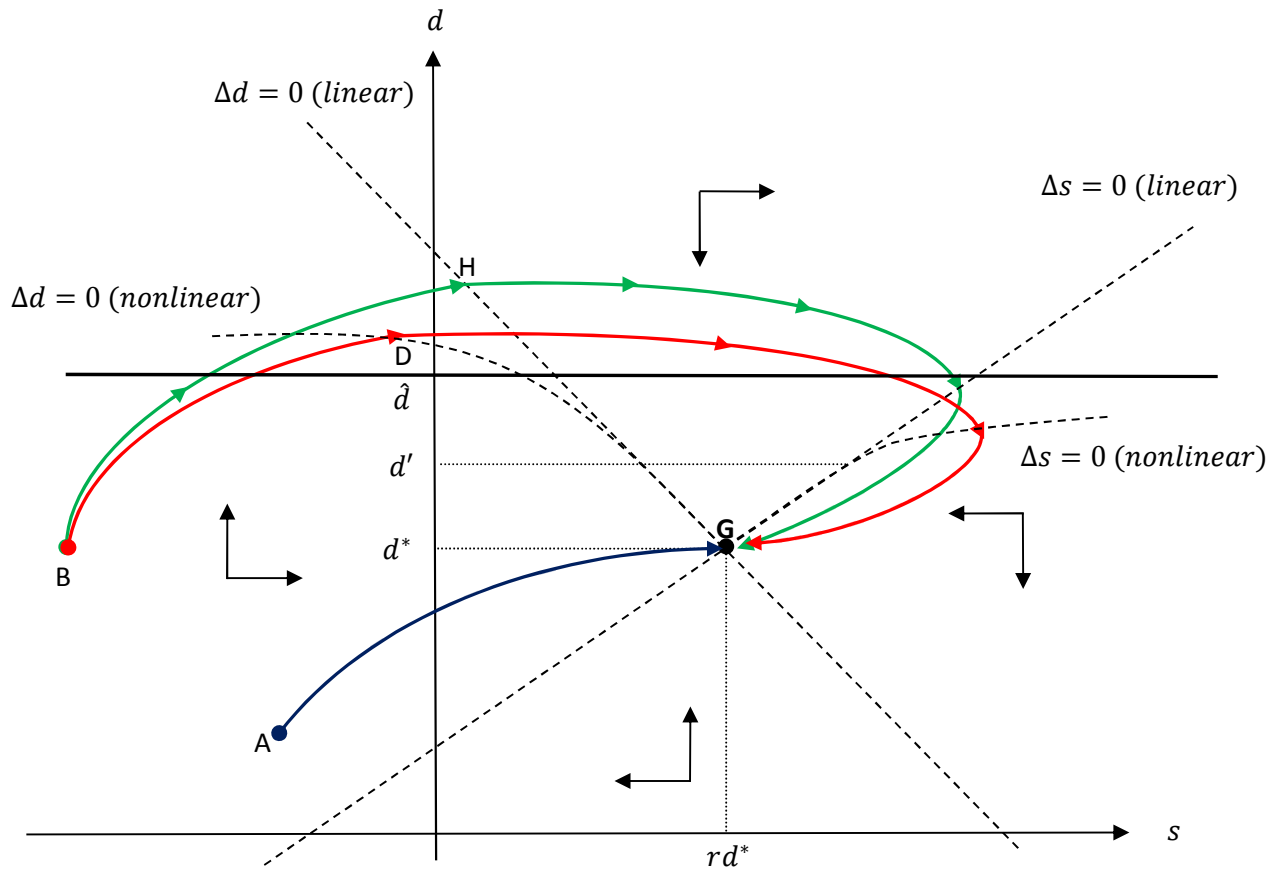


Figure 1: Nonexplosive adjustment paths under a linear (BHG) and a nonlinear (BDG) policy rule.



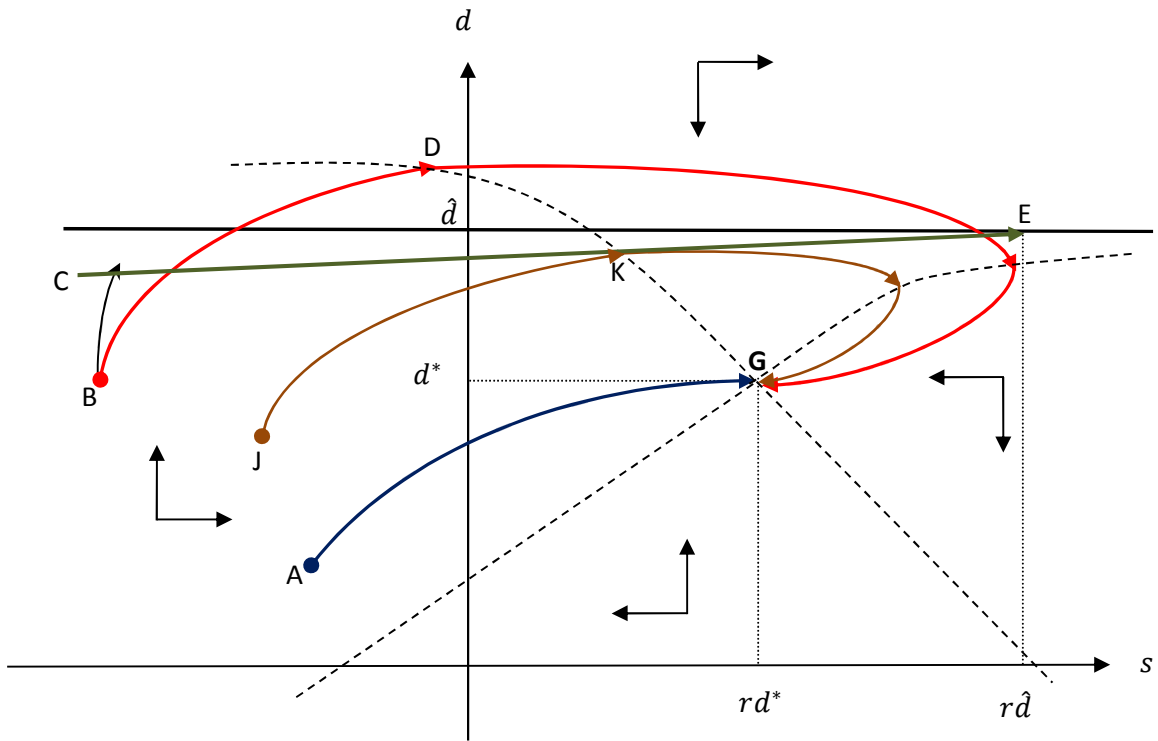


Figure 2: Policy Switching

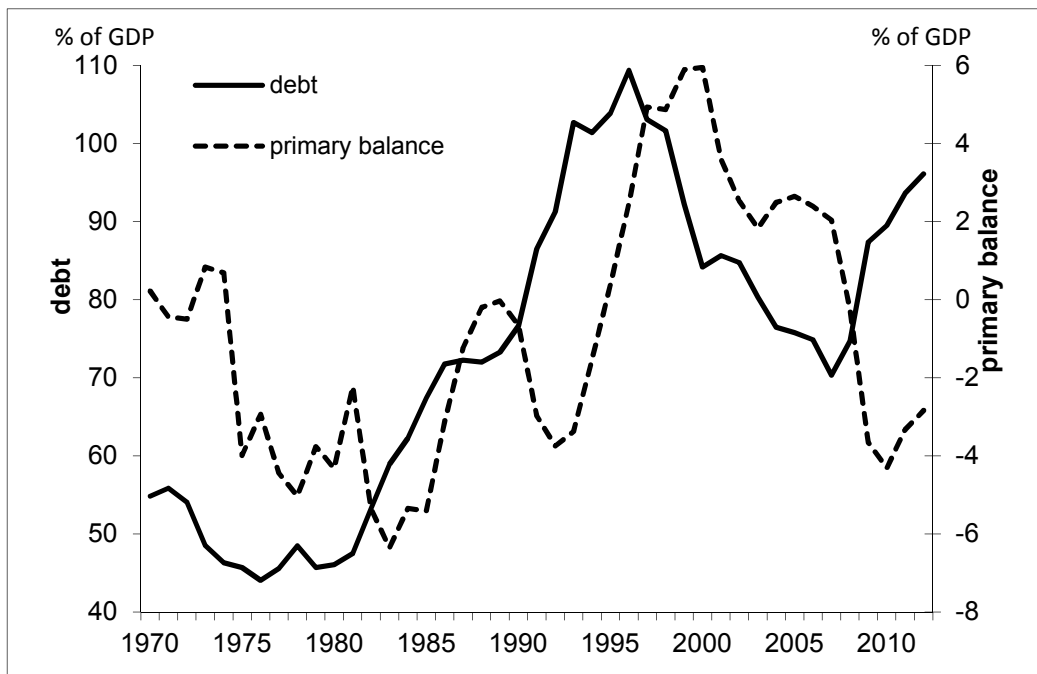


Figure 3: Gross debt and primary surplus relative to GDP.

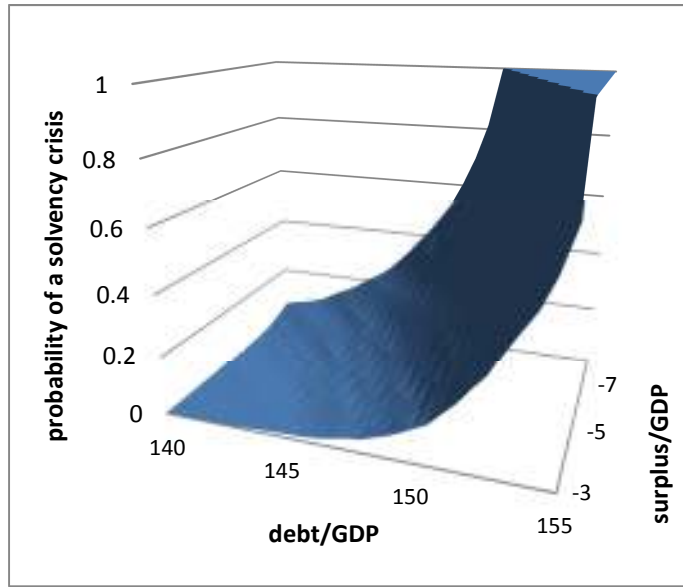


Figure 4: Probability of a solvency crisis as a function of debt/GDP and primary surplus/GDP

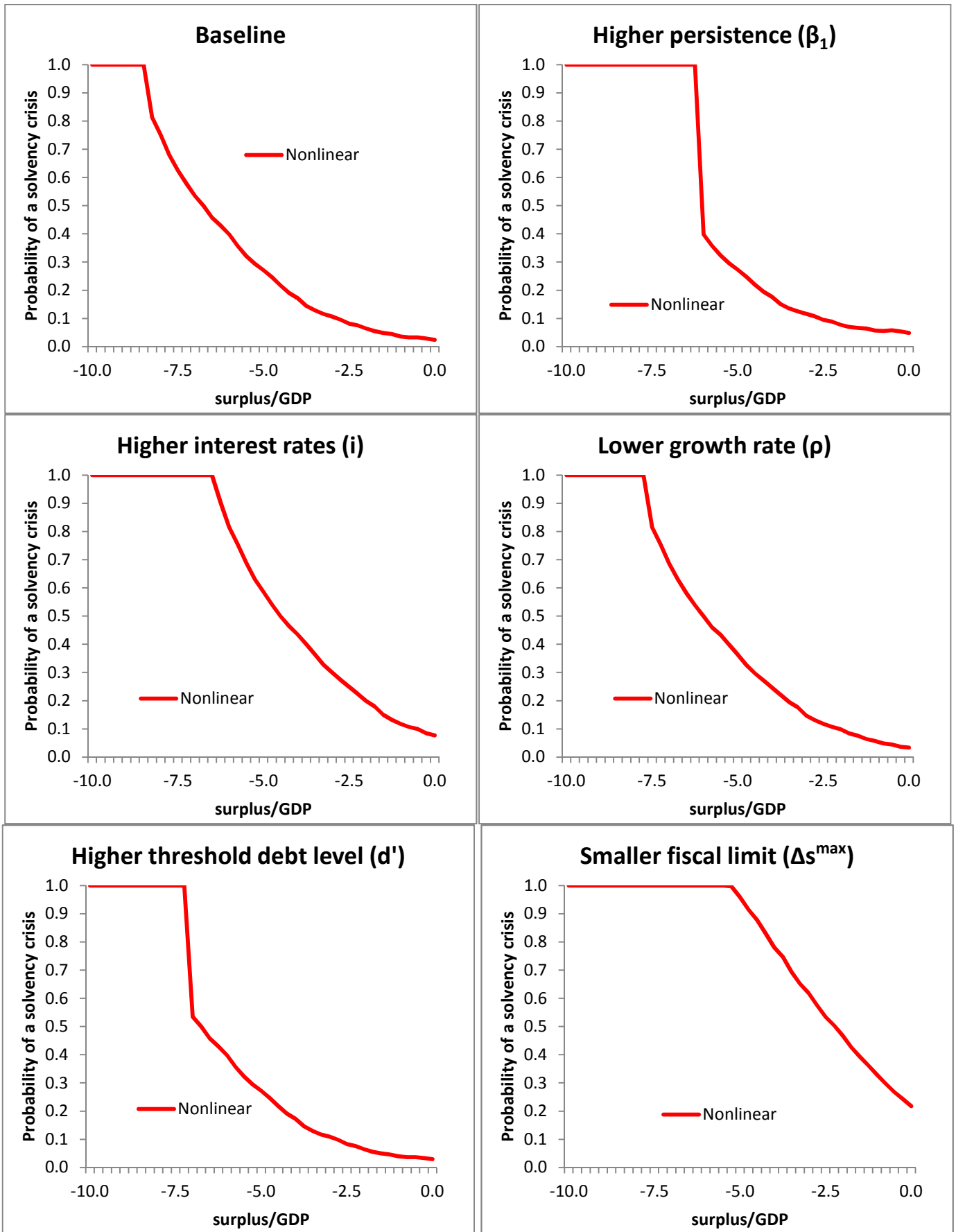


Figure 5: Probability of a solvency crisis as a function of primary surplus/GDP

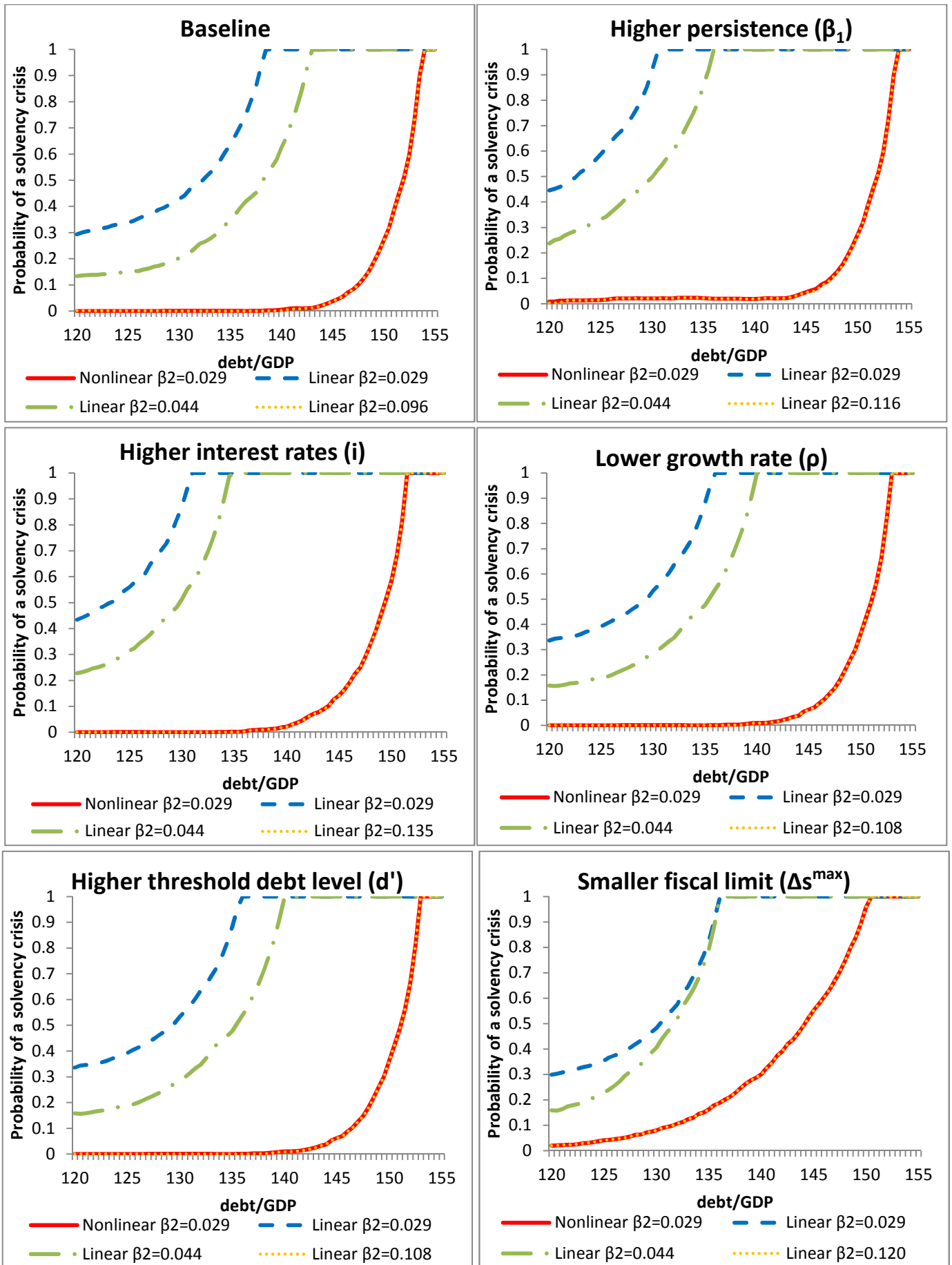


Figure 6: Probability of a solvency crisis as a function of debt/GDP

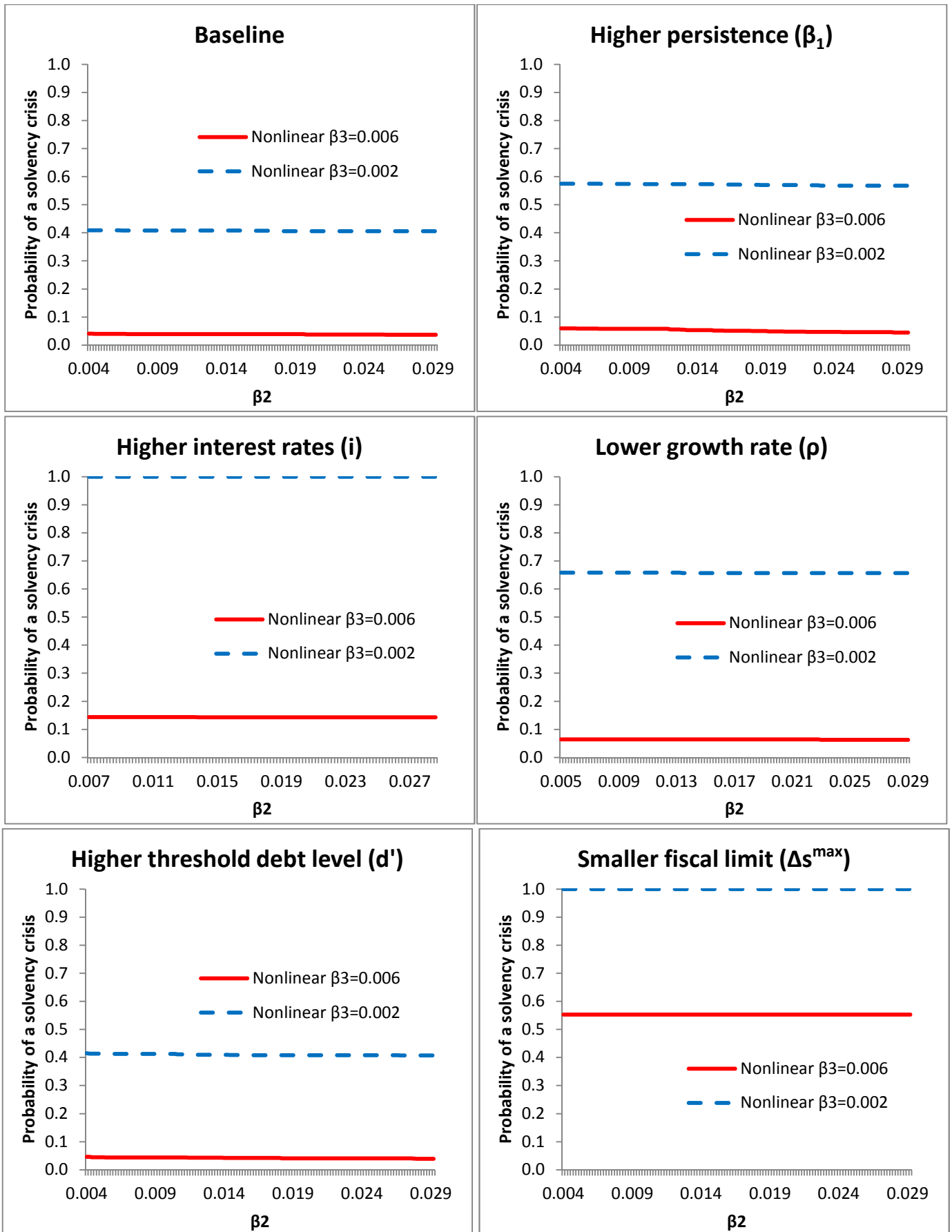


Figure 7: Probability of a solvency crisis as a function of  $\beta_2$